Efficient Power Allocation for Multiple Relays with Lossy Intra-Links and Distributed Source Coding

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Abstract—The so-called Chief Executive Officer problem has inspired some relay transmission schemes. It suggests that the source message can be recovered at the destination by combining a set of corrupted replicas sent by multiple relays, as long as the replicas are sufficiently correlated with the original message. In this work, we build on the Slepian-Wolf theorem to assess the outage performance of a distributed source coding scheme for a decode-and-forward multirelay system in which the direct link is unavailable to convey information. As in the CEO problem, the replicas forwarded by the relays are allowed to contain intra-link errors due to previous unreliable hops, and the destination is expected to reconstruct the original message by jointly decoding all the received replicas. In addition to analyzing the outage probability of such scheme, we derive a simple yet efficient power allocation strategy for the multiple relays, which is asymptotically optimal at high signal-to-noise ratio.

Index Terms—CEO problem, distributed source coding, outage probability, power allocation, relay channel, Slepian-Wolf theorem.

I. INTRODUCTION

In some abnormal scenarios, wireless networks may be faced with challenging requirements in terms of energy efficiency and reliable information transfer under a constantly changing network topology. For example, in case of severe environmental disasters, the communication infrastructure of mobile cellular networks may be seriously damaged, which in turn may lead to a collapse of the entire communication system. In such a scenario, intact mobile devices can be used to establish a mesh network without the need for central coordination and backbone infrastructure. However, the transmit power of mobile devices is inherently restricted by their limited power supply, so that low signal-to-noise ratios (SNRs) are likely to occur, resulting in an unreliable information transfer across the mesh network.

Cooperative communication techniques have emerged as a promising approach to support those restrictive transmissions, by exploiting the spatial diversity through a collection of intermediate relay nodes between source and destination [1]. Normally, before forwarding information, a relay node checks the received message and discard it if any uncorrectable error is detected, in order to ensure a reliable communication. However, in such mesh networks, intra-link errors between the source and each relay are common and inevitable, so that a huge amount of retransmissions would be needed to enable a reliable multi-hop connectivity.

In order to prevent energy inefficiency and improve the robustness of the referred network, an innovative distributed source coding (DSC) scheme was proposed in [2]. The DSC scheme exploits the fact that an erroneous message at the relay can be still highly correlated with the original source message and thus can assist in the decoding process at the destination—instead of being merely discarded. The performance gain of this approach can be reasoned with use of the Slepian-Wolf correlated source coding theorem [3]. In [2] and related studies, the link between the source and relay was considered to be lossy, being described as a binary symmetric channel (BSC) with a certain bit flipping probability [4]. This assumption is indeed a plausible amalgamated model for multiple unreliable hops that connect the source and relay. At the relay, a possibly erroneous message is detected, re-encoded, interleaved, and forwarded to the destination. Finally, at the destination, a joint decoding (JD) technique is applied to exploit the correlation between the original direct-link message and the unreliable relay message, by means of a likelihood ratio update function [2]. A significant improvement of the decoding performance is observed when compared with the relay message being discarded. In [5], a comprehensive outage analysis of the coding scheme proposed in [2] was carried out, by using the theorems for lossy source-channel separation and source coding with side information. In [6], the authors capitalized on the Slepian-Wolf theorem to analyze the outage probability of a correlated-source transmission scheme, assuming a decode-and-forward (DF) relay system with a BSC source-relay (SR) link and a relay-destination (RD) link under block Rayleigh fading. More recently, the same authors proposed in [7] a corresponding power allocation strategy to minimize the outage probability derived in [6].

In this work, based on the results in [6] and [7], we analyze the outage performance of a DSC scheme for a DF relay network with lossy intra-links, as well as we design an efficient power allocation strategy for the investigated scheme. However, differently from [6] and [7], we consider a more realistic scenario, in which no direct link exists between the source and destination. In addition, more generally than those works, we consider that each message is simultaneously transmitted along an arbitrary number of relay routes—instead of a single relay. It was shown in [8] that the joint decoder at

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the destination can exploit the correlation among the replicas received from multiple relays, and that a significant performance gain can be attained compared to conventional coding schemes. However, error-free retrieval of the original message cannot be achieved [9]. This is known as the Chief Executive Officer (CEO) problem in network information theory [10]. Similarly to [6], we capitalize on the Slepian-Wolf correlated source coding theorem to assess the outage performance of the investigated transmission scheme. This is a reasonable framework, since the various relays can be regarded as mutually correlated sources of information. Following this approach, we derive a remarkably simple closed-form asymptotic expression for the outage probability of the investigated system. More importantly, based on this expression, we derive a power allocation strategy that is asymptotically optimal at high SNR. To the best of our knowledge, Slepian-Wolf-based outage analysis and power allocation design for DSC relay networks with lossy intra-links have not been addressed yet in the context of multiple relays or unavailable direct transmission.

In what follows, \( P_r[\cdot] \) denotes probability, \( f_X(\cdot) \) is the probability density function (PDF) of a continuous random variable \( X \), \( p_Y(\cdot) \) is the probability mass function (PMF) of a discrete random variable \( Y \), \( H(Y) \) is the entropy of \( Y \), \( I(\cdot;\cdot) \) denotes mutual information, \( z \) is a sample realization of a generic random variable \( Z \), \( |S| \) is the cardinality of a set \( S \), \( B = \{0, 1\} \) is the binary set, \( \{A_i| i \in S\} \) is an indexed series (e.g., \( \{A_i| i \in \{1, 5, 7\}\} = \{A_1, A_5, A_7\} \)), and \( \varepsilon \) denotes “asymptotically equal to”.

II. SYSTEM MODEL

We consider a half-duplex dual-hop\(^1\) relay system as shown in Fig. 1. It consists of one source (S), one destination (D) and \( N \) DF relays (\( F_1, F_2, \ldots, F_N \)). The system operation is based on the CEO problem. An i.i.d. binary information sequence\(^2\) \( B_0 \) is originated by S with uniform probabilities \( \Pr[B_0 = 0] = \Pr[B_0 = 1] = 1/2 \). The source sequence is transmitted via \( N \) mutually independent BSCs with associated memoryless binary error sequences \( E_i \), \( i \in \{1, \ldots, N\} \)—a representation of the accumulated error caused by the multiple wireless hops up to the last one. The PMF of \( E_i \) can be written as

\[
p_{E_i}(e) = p_i \delta(e - 1) + (1 - p_i) \delta(e),
\]

(1)

\( e \in \{0, 1\} \), in which \( 0 < p_i \leq 0.5 \) is the bit-flipping probability and \( \delta(\cdot) \) is the discrete delta function. Therefore, the \( i \)th relay \( F_i \) observes an information sequence \( B_i = B_0 \oplus E_i \), with “\( \oplus \)” denoting the binary exclusive OR operation. Note that, like the source sequence, all relay sequences are also uniformly distributed, so that \( H(B_i) = 1, \forall i \). In addition, note that the relay sequences \( B_1, \ldots, B_N \) are mutually correlated. Those sequences are transmitted to D over independent channels undergoing flat Rayleigh fading (RF) and additive white Gaussian noise with mean power \( N_0 \). At the destination, the

\(^1\)As mentioned before, in our model the first hop is an amalgamated representation of possibly multiple hops between the source and relay.

\(^2\)In order to alleviate the notation, we shall drop the time index when denoting information and error sequences.

![Fig. 1: System model for the multirelay scheme based on the CEO problem.](image)

relay sequences are estimated as \( \hat{B}_1, \ldots, \hat{B}_N \) and, based on these, the source sequence is finally estimated as \( \hat{B}_0 \). The PDF of the received instantaneous SNR \( \Gamma_i \) at the \( i \)th second hop is exponentially distributed, thus given by

\[
\bar{f}_{\Gamma_i}(\gamma_i) = \frac{1}{\bar{\Gamma}_i} \exp(-\frac{\gamma_i}{\bar{\Gamma}_i}),
\]

(2)

where \( \bar{\Gamma}_i \) is the average SNR, obtained as

\[
\bar{\Gamma}_i = \frac{P_i}{N_0} \cdot d_i^{-\eta},
\]

(3)

with \( P_i \) being the transmit power at \( F_i \), \( d_i \) being the distance between \( F_i \) and \( D \), and \( \eta \) being the pathloss exponent.

III. PRELIMINARIES

In any effective JD scheme, the best recovery of the source message \( B_0 \) at the destination is obviously expected to be achieved when all recovered relay messages \( \hat{B}_1, \ldots, \hat{B}_N \) are error-free. However, even in such a favorable scenario at the second hops, the source error probability \( \Pr[\hat{B}_0 \neq B_0] \) cannot be zero, since the formulation of the CEO problem excludes error-free BSCs at the first hops (\( p_i > 0, \forall i \)). So far, only a few JD schemes for the referred problem have been proposed that exchange decoding information among the relays as a means to reduce the source error probability. In particular, the joint decoder proposed in [11] shows a significant performance gain when compared to other coding schemes. The exact calculation of the source error probability for the joint decoder in [11] is provided in [9].

In this work, we neither address any specific DSC/JD scheme nor perform any error probability calculations. Instead, following the approach in [6], we assess the outage performance limits of a generic DSC/JD scheme from an information-theoretical perspective. To this end, the joint decoder performance gain can be reasoned by means of the Slepian-Wolf correlated source coding theorem [3], because the multiple relay messages can be seen as correlated information sources, each of which resembles to some extent the original source message.

In its original scope, the Slepian-Wolf theorem provides the rate conditions for recovering all relay messages at the
destination. However, from an engineering perspective, this is not the primary aim. In the investigated system, the destination is ultimately not interested in recovering all relay messages—which are possibly erroneous replicas, indeed—but in merging them somehow to recover the original source message. Note that each relay sequence contains a different amount of information about the source sequence, depending on the channel quality of the corresponding first hop. To gain insight, let us consider two extreme sample cases. First, when the first hop is fully unreliable, i.e., if the bit-flipping probability equals 1/2, then the relay sequence cannot contain any useful information about the source sequence and thus should be discarded altogether. In such case, no rate constraint should be imposed at all. Second, when the first hop is fully reliable, i.e., if the bit-flipping probability is zero, then the relay sequence is identical to the source sequence. In such case, it would be desirable to fully recover the relay sequence and thus the rate constraint should be maximal. These two examples suggest that, in the general case, an appropriate rate requirement for a given relay should not depend on the absolute information content of the relay message (entropy), but on how much of this content concerns the source sequence (mutual information). This can be accomplished by adapting the Slepian-Wolf theorem accordingly. All in all, we propose modifying the original scope of the Slepian-Wolf theorem by replacing each entropy term with a corresponding mutual information term involving the source message. Specifically, the transmission rates \( R_i \) at the relays, \( i \in \{1, \ldots, N\} \), must satisfy the inequality constraints

\[
\sum_{i \in S} R_i \geq I (\{B_i| i \in S\}; B_0|\{B_j| j \in S^c\}) - I (\{B_j| j \in S^c\}; B_0) \tag{4}
\]

for all subsets \( S \subseteq \{1, \ldots, N\} \).

Hereafter, the set of \( N \)-tuples \( R_1, \ldots, R_N \) that satisfy all the constraints in (4) is referred to as the modified Slepian-Wolf admissible rate region. We now determine this region in terms of the bit-flipping probabilities of the first hops. This is paramount for the derivations that follow. Note in (4) that each constraint is written in terms of (i) the mutual information \( I (B_1, \ldots, B_N; B_0) \) between all relay sequences and the source sequence and (ii) the mutual information \( I (\{B_j| j \in S^c\}; B_0) \) between a subset of relay sequences and the source sequence. Any of these metrics can be evaluated as special cases of the following general formula:

\[
I (\{B_i| i \in S\}; B_0) = H (\{B_i| i \in S\}) - H (B_0, \{B_i| i \in S\}) + 1, \tag{5}
\]

where \( H (\{B_i| i \in S\}) \) is written as

\[
H (\{B_i| i \in S\}) = - \sum_{\{b_i\} \in B_S^i} \Pr (\{B_i| i \in S\} = \{b_i\}) \log_2 \Pr (\{B_i| i \in S\} = \{b_i\}), \tag{6}
\]

and \( H (B_0, \{B_i| i \in S\}) \) as

\[
H (B_0, \{B_i| i \in S\}) = - \sum_{\{b_0, \{b_i\}\} \in B_S^{i+1}} \Pr (\{B_0, \{B_i| i \in S\}\} = \{b_0, \{b_i\}\}) \log_2 \Pr (\{B_0, \{B_i| i \in S\}\} = \{b_0, \{b_i\}\}). \tag{7}
\]

The required probabilities \( \Pr (\{B_i| i \in S\} = \{b_i\}) \) can be obtained by knowing that the source bits are equally likely and by recognizing that both \( B_0 = 0 \) and \( B_0 = 1 \) may lead to each possible sample realization of \( \{B_i| i \in S\} \), which gives

\[
\Pr (\{B_i| i \in S\} = \{b_i\}) = \frac{1}{2} \prod_{i \in S} p_{E_i} (b_i) + \prod_{i \in S} \bar{p}_{E_i} (b_i), \tag{8}
\]

where we have used (i) the independence among the error sequences and (ii) the auxiliary PMF

\[
\bar{p}_{E_i} (e) \triangleq (1 - p_i) \delta (e - 1) + p_i \delta (e) \tag{9}
\]

defined by swapping the probabilities of \( E_i \). Similarly, \( \Pr ([B_i| i \in S] = \{b_i\}) \) can be obtained as

\[
\Pr ([B_i| i \in S] = \{b_i\}) = \frac{1}{2} \left( \delta (b_0) \prod_{i \in S} p_{E_i} (b_i) + \delta (b_0 - 1) \prod_{i \in S} \bar{p}_{E_i} (b_i) \right). \tag{10}
\]

Note that (8) and (10) are ultimately given in terms of the bit-flipping probabilities \( p_i \) associated with the first hops. Accordingly, using these into (6) and (7), and then into (5) and (4), we finally obtain the modified admissible rate region in terms of the bit-flipping probabilities.

IV. ASYMPTOTIC OUTAGE PROBABILITY

In the proposed system, an outage event occurs whenever the transmission rates \( R_1, \ldots, R_N \) fall outside the modified Slepian-Wolf admissible rate region. Such condition means that, at least for one of the relays, the information content regarding the source message cannot be entirely recovered at the destination. The maximum achievable value of \( R_i \) is related to the received SNR \( \Gamma_i \) by means of [5]

\[
R_i = \frac{1}{R_{ei}} \log_2 (1 + \Gamma_i), \tag{11}
\]

where \( R_{ei} \) represents the spectrum efficiency associated with the modulation and channel coding schemes [7]. In many parts of this work, for simplicity, we shall assume \( R_{ei} = R_{ei}, \forall i \). Using (11), each rate constraint in (4) that defines an outage event can be rewritten in terms of an equivalent SNR constraint. In this Section, we follow this approach to derive a simple and insightful closed-form asymptotic outage expression for the DSC scheme under investigation.

In principle, from (4) and (11), the outage probability can be evaluated by integrating \( f_{\Gamma_1} (\cdot) \cdots f_{\Gamma_N} (\cdot) \) over the SNR range corresponding to the modified inadmissible rate region. We have tried to solve this \( N \)-fold integral-form expression by splitting the integration region into smaller parts, but we ended up with an \((N - 1)\)-fold integral-form solution. On the other hand—and much more importantly to the power allocation...
scheme proposed next—, a remarkably simple closed-form asymptotic solution can be obtained at high SNR. This is based on the following key result of a pioneering work in [12]: the asymptotic outage behavior at high SNR is exclusively determined by the PDF behavior of the SNR in the vicinity of the origin. Therefore, in our case, it suffices to consider those parts of the modified inadmissible rate region that directly interface with at least one of the coordinate axes. Accordingly, from the Slepian-Wolf constraints given in (4), the outage probability \( P_{\text{out}} \) can be asymptotically formulated as

\[
P_{\text{out}} \approx 1 - \text{Pr}[R_1 > I(B_1; B_0|B_2, \ldots, B_N),
R_2 > I(B_2; B_0|B_1, \ldots, B_N), \ldots,
R_N > I(B_N; B_0|B_1, \ldots, B_{N-1})],
\]

\[
= 1 - \text{Pr}\left[2^{R_1 I(B_1; B_0|B_2, \ldots, B_N)} - 1 < \Gamma_1 < \infty,
2^{R_2 I(B_2; B_0|B_1, \ldots, B_N)} - 1 < \Gamma_2 < \infty, \ldots,
2^{R_N I(B_N; B_0|B_1, \ldots, B_{N-1})} - 1 < \Gamma_N < \infty\right].
\]

(12)

Now, by using the marginal PDFs given in (2) into (12) and by invoking the approximation \( \exp(x) \approx 1 - x, x \ll 1 \), after some algebraic manipulations an asymptotic high-SNR expression for the outage probability of the investigated system can be finally obtained as

\[
P_{\text{out}} \approx \sum_{i=1}^{N} \frac{C_i}{\Gamma_i},
\]

(13)

where each constant \( C_i \) is defined as

\[
C_i = 2^{R_i I(B_i; B_0|i \neq j)} - 1,
\]

(14)

with each mutual information \( I(B_i; B_0|i \neq j) \), \( i \in \{1, \ldots, N\} \), being computed from (4)–(10) in terms of the bit-flipping probabilities.

Note how (13) is extremely simple and compact as compared to the original \( N \)-fold integral-form representation. This is a major analytical contribution of this work.

V. ASYMPTOTICALLY OPTIMAL POWER ALLOCATION

In this Section, based on (13), we design a simple power allocation strategy for the investigated system in order to improve its outage performance. Despite its simplicity, the proposed allocation proves highly efficient, being asymptotically optimal at high SNR. For that reason, we call it Asymptotically Optimal Power Allocation (AOPA).

Given a total amount of transmit power \( P_T \) for all relays, the transmit power at the \( i \)-th relay is assigned as \( P_i = \alpha_i P_T \), where \( 0 \leq \alpha_i \leq 1 \) is the power allocation coefficient, \( i \in \{1, \ldots, N\} \). Of course, \( \sum_{i=1}^{N} \alpha_i = 1 \). Then, from (3), the average received SNR at the \( i \)-th second hop can be written as

\[
\bar{\Gamma}_i = \frac{\alpha_i P_T d^{-\eta}_i}{N_0}.
\]

(15)

Our primary aim is to find the set of power allocation coefficients \( \alpha_1, \ldots, \alpha_N \) that minimize \( P_{\text{out}} \), that is,

\[
\text{minimize} \quad P_{\text{out}}(\alpha_1, \ldots, \alpha_N)
\]

subject to \( 0 \leq \alpha_i \leq 1, \forall i, \) and \( \sum_{i=1}^{N} \alpha_i = 1 \).

(16)

Unfortunately, as seen in the previous Section, there exists no general exact closed-form expression for \( P_{\text{out}} \). Alternatively, we propose minimizing the simple asymptotic outage expression in (13). By using (3), this can be formulated as

\[
\text{minimize} \quad \sum_{i=1}^{N} \frac{N_0 C_i d_i^p}{P_T} \cdot \frac{1}{\alpha_i}
\]

subject to \( 0 \leq \alpha_i \leq 1, \forall i, \) and \( \sum_{i=1}^{N} \alpha_i = 1 \).

(17)

where each constant \( C_i \) is defined as in (14). This is a convex optimization problem, as follows. Note that the cost function is a summation, each component of which is a function of a single power allocation coefficient. It turns out that the \( i \)-th component \( N_0 C_i d_i^p / (P_T \alpha_i) \) is a convex function of the \( i \)-th coefficient \( \alpha_i \), because \( N_0 C_i d_i^p / P_T \geq 0 \) and \( 1/\alpha_i \) is a convex function of \( \alpha_i \). The proof of convexity is completed by recognizing that a sum of convex functions is also a convex function [13]. To find its global minimum, we eliminate the \( N \)-th power allocation coefficient \( \alpha_N \) by incorporating the constraint \( \sum_{i=1}^{N-1} \alpha_i = 1 \) into the cost function, which gives

\[
\sum_{i=1}^{N-1} \frac{N_0 C_i d_i^p}{P_T} \cdot \frac{1}{\alpha_i} + \frac{N_0 C_N d_N^p}{P_T} \cdot \frac{1}{1 - \sum_{i=1}^{N-1} \alpha_i}.
\]

(18)

Then, by differentiating (18) with respect to the remaining set of power allocation coefficients \( \alpha_1, \ldots, \alpha_{N-1} \), by equating all these partial derivatives to zero, and by solving the resulting system of equations, after some algebraic manipulations omitted here for simplicity, we finally arrive at the AOPA scheme:

\[
\alpha_i = \frac{C_i \cdot d_i^p}{\sum_{j=1}^{N} C_j \cdot d_j^p}, \quad i \in \{1, \ldots, N\}.
\]

(19)

Note that the proposed power allocation depends ultimately on the distances \( d_i \) between each relay and the destination, and on the conditional mutual information \( I(B_i; B_0|i \neq j) \) between each relay sequence and the source sequence, given in (4)–(10) as a function of the bit-flipping probabilities.

The power allocation scheme in (19) is the main contribution of this work.

VI. NUMERICAL RESULTS

In this Section, we evaluate the impact of our AOPA policy on the performance of the investigated DSC scheme, by considering some representative sample scenarios. The equal power allocation (EPA) policy, i.e., \( \alpha_i = 1/N, \forall i \), is included for comparison. In each scenario, the outage probability is assessed asymptotically, by means of (13), as well as via Monte Carlo simulation. For illustration purposes, we assume a binary phase-shift keying modulation and a channel code
TABLE I: Bit-flipping probabilities, AOPA coefficients, and AOPA-over-EPA SNR gain for investigated scenarios.

<table>
<thead>
<tr>
<th>Scen.</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>Gain (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
<td>0.1</td>
<td>—</td>
<td>0.824</td>
<td>0.176</td>
<td>—</td>
<td>1.54</td>
</tr>
<tr>
<td>2</td>
<td>0.001</td>
<td>0.3</td>
<td>—</td>
<td>0.983</td>
<td>0.017</td>
<td>—</td>
<td>2.88</td>
</tr>
<tr>
<td>3</td>
<td>0.001</td>
<td>0.01</td>
<td>0.04</td>
<td>0.601</td>
<td>0.225</td>
<td>0.174</td>
<td>1.29</td>
</tr>
<tr>
<td>4</td>
<td>0.001</td>
<td>0.02</td>
<td>0.3</td>
<td>0.797</td>
<td>0.157</td>
<td>0.046</td>
<td>3.12</td>
</tr>
</tbody>
</table>

rate of 1/2, so that $R_c = 2$. Moreover, we assume $\eta = 4$ and a half-normalized distance $d_1 = 0.5$ between the relays and destination. We illustrate the system performance for two and three relays, by exploiting multiple configurations in terms of bit-flipping probabilities and average SNR. The investigated scenarios are listed in Table I along with the power allocation coefficients provided by AOPA and the corresponding SNR gains with respect to EPA. Fig. 2 shows the outage probability versus the average system transmit SNR for each scenario. The following can be observed from the curves: (i) our asymptotic expression in (13) gives an excellent match at medium to high SNR; (ii) in all the cases, AOPA outperforms EPA at medium to high SNR; and (iii) the more dissimilar are the bit-flipping probabilities of the first hops, the greater is the SNR gain achieved by AOPA when compared with EPA. The last observation is indeed expected, by considering that, for identically distributed first hops and identical relay-to-destination distances, the AOPA coincides with EPA, then providing no gain at all.

VII. CONCLUSIONS

We analyzed the outage performance of a distributed source coding scheme in a multirelay system with intra-link errors. More importantly, we designed a simple and highly effective power allocation scheme for this system. Our results find important applications in emerging links-on-the-fly technologies for robust and efficient communications in unpredictable environments. It is noteworthy that we have preliminarily tested our power allocation strategy into practical DSC/JD schemes, as that proposed in [11]. Strikingly, in all the tests, the observed error-rate performance was nearly optimal. This issue shall be further investigated in forthcoming submissions.

REFERENCES