Reliability Assessment of Fault Tolerant Routing Algorithms in Networks-on-Chip: An Analytic Approach

Sadia Moriam*† and Gerhard P. Fettweis *†

*Dep. of Electrical Engineering and Information Technology / Vodafone Chair Mobile Communication Systems
†Centre for Advancing Electronics Dresden (CFAED)
Technische Universität Dresden, 01062 Dresden, Germany
Email: {sadia.moriam, gerhard.fettweis}@tu-dresden.de

Abstract—Rapid scaling of transistor gate sizes has significantly increased the density of on-chip integrations and paved the way for many-core systems-on-chip with highly improved performances. The design of the interconnection network of these complex systems is a critical one and the network-on-chip is now the accepted efficient interconnect for such large core arrays. An unfortunate adverse effect of technology scaling is the increased susceptibility to failures resulting in failing links and routers in the network-on-chip. To keep the network connected, efficient fault adaptive routing algorithms are necessary to route around faults. To design and evaluate the fault resiliency of such adaptive routing algorithms, fast, accurate and flexible analytic models are required, especially in large networks for which simulations are extremely time costly. In this paper, we present an analytic approach to evaluate the reliability of adaptive routing algorithms based on algebraic manipulations of the channel dependency matrix. It allows also to evaluate the number of alternate paths between source-destination pairs, in the presence of any number of permanent faults in the network. The analytic model is general and can be adapted to evaluate network reliability for any network topology and with any adaptive routing algorithm based on the turn model. We present cycle-accurate simulations to compare the accuracy of the model for the 2-D mesh and the hexagonal networks. The model is able to estimate the network fault resilience with an accuracy of about 1% and more than 70 times faster than the cycle accurate simulation.

I. INTRODUCTION

With the ever increasing need for higher on-chip computing power, the evolution towards many-core heterogeneous systems with hundreds to thousands of processing cores is considered inevitable [1]. The performance of such large heterogeneous systems would be limited by the traditional bus-based connection network. As a result, over the last decade Network-on-Chip (NoC) has emerged as the highly scalable and efficient interconnection network for many-core systems [2]. To design reliable and efficient NoCs, a number of factors must be taken into account such as the interconnection topology, the routing algorithm, employed switching method etc. Parameters such as the average packet latency, throughput, reliability etc. are some of the important NoC performance parameters.

Due to area and energy restrictions, NoCs mainly use wormhole switching inside routers, with advantages such as reduce buffer sizes and latency. In wormhole switching [3], the message is divided into smaller packets called flits, and the flits are transferred immediately one after the other from the source to the destination. The entire path is reserved until the last or tail flit passes. However, deadlock problems arise when using adaptive routing for traffic balance and/or fault-tolerance. Deadlocks occur due to a cyclic dependency of blocked resources, resulting in the flits in the deadlocked channels being blocked indefinitely as no flits caught in the deadlock cycle can progress forward. Deadlock problems are usually avoided by the routing algorithm by preventing the reservation of channels in a cyclic manner.

Technology scaling has increased the susceptibility of NoC components to failures [4]. Redundancy is the general approach to fault tolerance, with different forms of redundancy to tackle the different classes of faults [4]. Spatial or modular redundancy i.e. the use of redundant system components is one approach for tolerating permanent router and link faults. In addition, fault-adaptive routing algorithms must be employed to bypass faults. The commonly used NoC interconnection topology is the mesh, which already provides redundant pathways. The hexagonal NoC topology having diagonal links between routers provides even greater path diversity making it more fault resilient. The general approach for analyzing the performance of adaptive routing algorithms for NoC is by lengthy cycle-accurate simulations usually done for 100,000 or more cycles depending on the network size. To evaluate fault-adaptive routing algorithms, faults (on links or routers or both) are injected at random locations over 10,000 or more iterations and then the results are averaged. Fault resilience is given by the ratio of successfully received flits to total number flits injected into the network.

In this paper, we propose an approach to evaluate analytically the network resilience with fault-tolerant Negative-First adaptive routing algorithm for the mesh and hexagonal NoCs. The analytic approach determines whether a path connects a source-destination node pair as provided by the algorithm, in the presence any number of random router faults. The overall NoC fault resilience is then calculated by averaging of over all
source-destination pairs as dictated by the traffic scenario. Although, we consider only router faults in this paper, the model is very general and can easily include effect of link faults as well. Moreover, the approach is very flexible and any other routing algorithm can also be easily modeled. Our approach is significantly faster than cycle-accurate simulations, making it very useful for analyzing larger network sizes efficiently.

The next sections are organized as follows: Section II discusses some related works and in Section III adaptive routing and the turn model is discussed. The reliability evaluation approach is discussed in section IV and the application to mesh and hexagonal NoC is given in section V. Section VI evaluates the analytic model in comparison to simulation results. Section VII concludes the paper and discusses some future works.

II. RELATED WORKS

Most investigations of fault-tolerant routing for NoC, such as [5] [6] are based on cycle-accurate simulations. Due to the challenges of accurately modeling adaptive routing algorithms, there are fewer works in this category. An analytic model was given in [7] to evaluate the reliability of XY and XY-YX mesh NoC based on the average path length calculation. The authors also propose extensions of their analysis to adaptive routing algorithms such as the west-first routing algorithm. A fault-tolerance analysis of different NoC architectures was presented in [9] based on reliabilities of the different components such as the routers and NoC interfaces (NI). All combinations of different number of inoperable routers and NIs were investigated and the overall system reliability was obtained by summing up the reliabilities of all the combinations. With simple source-based routing, it was found that network structures built from simple 3-port routers provided better fault tolerance than those based on more complex multi-port routers. Refan et al. [10] determine NoC reliability for application specific traffic with XY routing, when considering router failures. The path reliability for a source-destination pair is obtained by considering the product of the reliabilities of all routers in the path. The dependence of Through-silicon-via (TSV) failures on 3-D NoC reliability has been explored by analytic method in [11]. There the authors modeled the TSV characteristics as a time-invariant failure probability and quantified the relationship between NoC reliability and TSV failure.

III. ADAPTIVE ROUTING AND TURN MODEL

The adaptive routing algorithms whose reliability are assessed analytically in this paper are based on the turn model. The turn model [12] is a popular technique for developing deadlock-free adaptive routing algorithms without the usage of costly virtual channels. Accordingly, a turn happens when a flit changes its direction of travel. Deadlock cycles are created when flits reserving some channels want to make a turn, but cannot do so because the requested channel is held by some other flits. Accordingly, closed cycles of dependencies are created and due to the deadlock, no flits can move forward. To prevent deadlock, enough turns should be avoided both in the clockwise (CW) and counter clockwise (CCW) directions to prevent the dependency cycle.

For the mesh, a total of four turns in the CW direction or in the CCW direction contribute to cycles, so that one turn must be prevented in CW and CCW directions to prevent deadlock. Accordingly, three different adaptive routing algorithms are possible for the mesh. Of these we concentrate on the Negative-First routing algorithm [12] in which any turn from the positive to the negative directions are prevented i.e. any turn from north or east directions to the west or south directions. A fault tolerant version of this routing algorithm was presented in [13] and is described briefly in Table I. It considers router faults only and is modeled with our analytic approach in the following section V. It should be noted that for both the mesh and hex, we consider the algorithm with a certain level of non-minimality although further non-minimal versions are possible.

### TABLE I

<table>
<thead>
<tr>
<th>Destination</th>
<th>N</th>
<th>E</th>
<th>S</th>
<th>W</th>
<th>local</th>
</tr>
</thead>
<tbody>
<tr>
<td>NE</td>
<td>N,E*</td>
<td>N,E*</td>
<td>N,E*</td>
<td>N,E*</td>
<td>N,E*</td>
</tr>
<tr>
<td>E</td>
<td>E</td>
<td>E</td>
<td>S,E</td>
<td>-</td>
<td>S,E</td>
</tr>
<tr>
<td>N</td>
<td>N</td>
<td>N</td>
<td>-</td>
<td>W</td>
<td>N W</td>
</tr>
<tr>
<td>NW</td>
<td>-</td>
<td>-</td>
<td>W,S</td>
<td>W,S</td>
<td>W,S</td>
</tr>
<tr>
<td>SE</td>
<td>-</td>
<td>-</td>
<td>S,W</td>
<td>S,W</td>
<td>S,W</td>
</tr>
<tr>
<td>S</td>
<td>-</td>
<td>-</td>
<td>W,S</td>
<td>W,S</td>
<td>W,S</td>
</tr>
<tr>
<td>SW</td>
<td>-</td>
<td>-</td>
<td>W,S</td>
<td>W,S</td>
<td>W,S</td>
</tr>
</tbody>
</table>

*Select so that source and destination are not in one line

Application of the turn model to the the hexagonal NoC, created by addition of diagonal links in the mesh NoC produces similar fault-tolerant algorithms, as given in [14]. For the negative first routing algorithm for the hex NoC, all turns from the positive directions i.e. North(N), East(E) and North East(NE) to the negative directions i.e. South(S), West(W) and South West(SW) directions are prevented.

IV. SYSTEM MODEL AND MATRIX ALGEBRA

A. Connectivity matrix

Graph theory is a very useful tool for description and property evaluation of networks. In general, a network consisting of \( N \) vertices or nodes can be represented by a graph \( G = (V, E) \), where \( V \) is the set of vertices \( (v_1, v_2, ..., v_N) \) and \( E \) is the set of edges connecting the vertices. If the connections between the nodes are directed, the graph is called a directed one. The adjacency matrix is an \( N \times N \) matrix \( A = (d_{ij}) \) in which \( d_{ij} = 1 \) if there is an edge connecting vertices \( v_i \) and \( v_j \), otherwise this value is 0. Powers of the adjacency matrix gives the number of available paths between pairs of nodes. Thus, as given in [15], the ij-th entry of \( A^2 \) gives the number of paths from \( v_i \) to \( v_j \) using 2 hops:

\[
(A^2)_{ij} = \sum_{k=1}^{N} A_{ik} \cdot A_{kj}. \tag{1}
\]

Continuing with higher powers of \( A \), \( (A + A^2 + A^3)_{ij} \) would give the total number of paths from \( v_i \) to \( v_j \) using one, two or three edges or hops. Similarly, the following infinite series can be used for the presence of any path connectivity between any node pair, using any number of hops:

\[
A_{conn} = A + A^2 + ... + A^n + A^{n+1} + ... \tag{2}
\]
The matrix $A_{\text{conn}}$ is referred to as the node connectivity matrix. $A_{\text{conn}}$, as given above, cannot be used to give the connectivity of a wormhole switched network such as the NoC, since it does not take into account the turns restricted for deadlock prevention. Instead we consider the channel adjacency matrix also called the channel dependency matrix (CDM). The entries of the CDM, $A_{ij}$ represents the dependency of a channel $i$ to channel $j$, i.e. whether a flit can be forwarded from channel $i$ to channel $j$. $A_{ij}$ equals 1 iff $i$ and $j$ are adjacent and if channel $j$ can be requested immediately after channel $i$. The CDM for $2 \times 2$ mesh NoC with Negative-First routing is shown in Fig. 1. Here, since the channels of the NoC are directed, the total number of channels are 8: $c_1, c_2, \ldots, c_8$.

![Fig. 1. The channel dependency matrix (CDM), A for $2 \times 2$ Mesh NoC with Negative-First routing.](image)

As negative-first routing with the aforementioned turn restrictions is used, it is seen e.g. that $A(c_1, c_1 : c_8) = 0$, as this is a east directed output link, from which no flit can be forwarded to the south ($c_3$) or west ($c_4$) links, although they are adjacent physically. However, $A(c_6, c_7) = 1$ or $A(c_5, c_1) = 1$ as these do not break the turn restrictions.

**B. CDM Algebra with adaptive routing**

Considering the CDM, $A^n$ gives the number of paths from a channel $c_i$ to channel $c_j$ via $n+1$ intermediate channels taking into consideration the restrictions for avoiding deadlock. It should be noted that $A^2$ would mean the number of paths using 3 channels or hops and similarly $A^n$ refers to $n + 1$ hops. To get the number of paths from a node to another node, the entries of the matrix from all the output channels of the source node to all the input channels of the destination router have to be considered in the calculation. A small example for a $4 \times 4$ mesh NoC is given in Fig. 2.

![Fig. 2. Powers of the CDM, A and the channel connectivity matrix, Chh with Negative First routing.](image)

When there is permanent fault on a link, all dependencies from and to this channel are made equal to zero, to remove its effect from the path calculations. If a router is permanently faulty, then all the dependencies from and to the output and input channels respectively of the faulty router are made equal to zero. As an example, if the router 11 fails completely, then $A(\bar{\sigma}, ;)=0$ and $A(; \bar{\tau})=0$, where $\tau$ and $\bar{\tau}$ are the group of output and input ports of the faulty router respectively. The effect on the path connectivity of node 14 to node 7 can be seen in Fig. 3, where node 7 is reachable only via $i_E$.

**V. ANALYTIC MODEL FOR NOC ROUTING CONNECTIVITY**

Analytically modeling node connectivity when using a particular routing algorithm in the presence of unlimited faults in the NoC is a challenging task due to the complexity of adaptive routing. Even when a physical path may exist, as allowed by the deadlock freedom constraint, whether a flit will actually reach the destination will depend upon whether the routing algorithm is able to find this path. NoC routers do not have a global view of the faults in the network. To keep the overhead minimum, a router usually has knowledge of faults of only the adjacent connected neighbor routers. As a result, in the presence of multiple faults and with distributed routing, it is not always possible to calculate the connectivity directly using the channel adjacency matrix.

For this reason, we break down the reliability assessment in two parts, first by determining whether the source node is connected to an intermediate node (called 'q' in the following). If this intermediate node is connected to the destination, then the source under consideration is also connected to the destination. The location of the intermediate node depends upon the routing algorithm. For certain destination directions e.g those to the E, N and for some destinations to the NE, we
can directly determine the connectivity, as shown below. For destinations to NW, SE, S, W and SW, we have to consider the intermediate node approach. This is because, e.g. to go to the NW, a flit should go first in the W or S direction until the destination is to the N of current router and then follows the rules for N-routing. Closer s-d pairs are calculated first and then farther s-d pairs. Consequently, the calculation time taken by the analytic approach will increase with the network size. However, it is still significantly less than that with simulation. Moreover, although we consider the negative-first routing algorithm only, other turn model algorithms can be similarly modeled by this approach. A short overview of the parameter symbols used in the analytic model is given in Table II.

**Table II**

**Overview of symbols used in analytic model**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>Source node</td>
</tr>
<tr>
<td>d</td>
<td>Destination node</td>
</tr>
<tr>
<td>q</td>
<td>Intermediate node</td>
</tr>
<tr>
<td>(o_H)</td>
<td>Output port of source node in H direction</td>
</tr>
<tr>
<td>(i_H)</td>
<td>Input ports of router</td>
</tr>
<tr>
<td>(Ch_h)</td>
<td>Channel connectivity matrix (ChH \times A^x)</td>
</tr>
<tr>
<td>(\Delta x)</td>
<td>x-distance between source and destination</td>
</tr>
<tr>
<td>(\Delta y)</td>
<td>y-distance between source and destination</td>
</tr>
<tr>
<td>(\Delta q)</td>
<td>distance between source and intermediate node (q)</td>
</tr>
<tr>
<td>(f_h)</td>
<td>Fault condition of output port in H direction</td>
</tr>
<tr>
<td>(P_{u,v})</td>
<td>Path connectivity from node (u) to node (v)</td>
</tr>
</tbody>
</table>

**Destination to the NE or N of source**

In this case, routing is done along the N and E adaptively towards the destination. If \(\Delta x = 1\) or \(\Delta y = 1\) (as shown in Fig. 4 (a),(b)), we can directly calculate the node connectivity from the connectivity matrix, \(Ch_h\) by summing over the matrix entries from \(o_N\) and \(o_E\) to the input ports, \(i_d\):

\[
P_{s,d} = \sum_{i_d} \sum_{t=o_N,o_E} Chh_{t,i_d}.
\]  

(3)

If \(\Delta x = \Delta y\), the E port is the preferred output port to the destination. If this faulty \((f_E)\), then the N port is used. If \(\Delta x = \Delta y \leq 3\):

\[
P_{s,d} = \sum_{i_d} Chh_{oE,i_d} + f_E \cdot \sum_{i_d} Chh_{oN,i_d}.
\]  

(4)

When \(\Delta x = \Delta y > 3\) (Fig. 4(c)), if the E port is non-faulty then the first node to the East-North that is reachable from \(s\) (i.e. \(A^{\Delta q-1}(o_E, i_q) > 0\)) is the intermediate node \(q\). If E-port is faulty, the N-port is selected:

\[
P_{s,d} = \sum_{i_d} Chh_{oE,i_q} \cdot P_{q,d} + f_E \cdot \sum_{i_d} Chh_{oN,i_q} \cdot P_{q,d}.
\]  

(5)

This equation is also applicable when \(\Delta x > \Delta y \neq 1\) (Fig. 4(d)). Here, the flit moves first to the E until \(\Delta x = \Delta y\), then the rules for this situation is followed. Thus \(q\) is the farthest node that can be reached (\(A^{\Delta q-1}(o_E, i_q) > 0\)) in the E direction without encountering a fault until \(\Delta x = \Delta y\). If E port is faulty, then the first node in the North East direction to be reached is \(q\). If \(\Delta x > \Delta y\) instead, then N direction is the preferred direction.

**Destination to the E or N of source**

For destination to E, if \(s\) and \(d\) are not along the south edge, then a routing to the W is done and then rules of NE-routing
are followed (Fig. 4(e)). If W-port is faulty, then E-port is used. Thus the reachability of \( d \) can be determined by:

\[
P_{s,d} = \sum_{i_d} A_{\delta s,i_d}^{\Delta x+1} + f_S \cdot \sum_{i_d} A_{\delta E,i_d}^{\Delta x-1}.
\]

If \( s \) and \( d \) are along the south edge and a faulty router blocks the path, then the flit is routed one hop perpendicular to the edge (Fig. 4(t)). Although, a restricted turn (E-to-S) is taken, no deadlock occurs as the cycle through a faulty edge router cannot be completed. The reachability of \( d \) is given by:

\[
P_{s,d_{-edge}} = \sum_{i_d} C_{\delta E,i_d} + f_E \cdot \sum_{i_d} C_{\delta N,i_d}.
\]

Similarly for destinations to the N, the following expressions are applicable:

\[
P_{s,d} = \sum_{i_d} A_{\delta W,i_d}^{\Delta y+1} + f_W \cdot \sum_{i_d} A_{\delta N,i_d}^{\Delta y-1}.
\]

\[
P_{s,d_{-edge}} = \sum_{i_d} C_{\delta N,i_d} + f_N \cdot \sum_{i_d} C_{\delta E,i_d}.
\]

Due to lack of space, expressions of reachability for other destinations could not be given here. But similar expressions are derived by considering the path taken according to the routing algorithm.

### A. Hex fault-tolerant routing algorithm

For the hexagonal NoC has 3 directions in the positive direction (E,NE and N) and in the negative directions (W,SW and S). As a result, there are 3 possible directions for reaching a destination. The routing algorithm is given in Table III. Due to lack of space, only the expression for destination to NE, \( \Delta x = \Delta y \) (Fig.5) is given here.

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>HEX FAULT Tolerant Negative-First Routing algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dest</td>
<td>N</td>
</tr>
<tr>
<td>NE</td>
<td>NE,NE,E*</td>
</tr>
<tr>
<td>E</td>
<td>E</td>
</tr>
<tr>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>NW</td>
<td>-</td>
</tr>
<tr>
<td>SE</td>
<td>-</td>
</tr>
<tr>
<td>S</td>
<td>-</td>
</tr>
<tr>
<td>NW</td>
<td>-</td>
</tr>
<tr>
<td>SW</td>
<td>-</td>
</tr>
</tbody>
</table>

Select so that source and destination are not in one line

| ** \( \Delta x = 1 \) or \( \Delta y = 1 \) |

If \( \Delta x = \Delta y \leq 3 \), then

\[
P_{s,d} = \sum_{i_d} \sum_{i_q=\delta E,i_q}^{\delta N} C_{h,i_q}.
\]

When \( \Delta x = \Delta y > 3 \), if \( A_{\delta N,i_d}^{\Delta x-1} < 0 \), then the last node in NE to be reached is q. If the NE router is faulty, E port is taken and the first node to the E-NE that is reachable from \( s \)}

![Fig. 5. (A) The hexagonal NoC, with the hexagon highlighted (b) Routing to the NE.](image)

\( (i.e.A_{\delta N,i_q}^{\Delta q-1} > 0) \) is the intermediate node \( q \). If E-port is faulty, the N-port is selected:

\[
P_{s,d} = \sum_{i_q} C_{\delta N,i_q} \cdot P_{q,d} + f_E \cdot \sum_{i_q} C_{\delta E,i_q} \cdot P_{q,d} + f_E \cdot \sum_{i_q} C_{\delta N,i_q} \cdot P_{q,d}.
\]

**VI. PERFORMANCE EVALUATION**

For comparing the result of the analytic approach, we carried out simulations for over 100,000 to 200,000 cycles in a cycle accurate simulator [16] (C/C++ based). Fault-tolerant negative-first routing algorithm was implemented for both the mesh and hexagonal NoC. The traffic considered was uniform random. Permanent router faults were injected into the NoC at random locations over 10,000 iterations. Fault-resilience was determined as the average ratio of the number of successfully received flits to the total number of injected flits. Similarly, using the analytic model the network fault-resilience i.e. the average node connectivity was calculated over 10,000 iterations of random fault locations and compared to the simulation results. Fig.6 shows the fault-resilience vs. the percentage of faults for \( 8 \times 8 \) mesh NoC. As can be seen, the model results match closely that of the simulation. The model is able to calculate the fault-resilience with accuracy of about 1%. Similar curves were plotted for the hexagonal NoC with

![Fig. 6. Results of fault resilience vs. percentage faulty routers for \( 8 \times 8 \) Mesh NoC in comparison to cycle-accurate simulation.](image)
negative-first fault tolerant routing. To test the scalability of the analytic approach, different NoC sizes of $6 \times 6$, $8 \times 8$, and $10 \times 10$ were simulated using both the cycle-accurate simulator and the model. The results, depicted in Fig.7 illustrate again the accuracy of the analytic approach. It can also be observed that the $8 \times 8$ hex NoC has higher fault resilience in comparison to the mesh of the same size. Moreover, as the size of the network increases, the network resilience decreases for the same percentage of faulty routers. This is the consequence of the higher average path length in the bigger network which increases the probability of encountering a fault. The duration of the cycle-accurate simulations became excessively long as the network size increased. For the $10 \times 10$ network, the analytic approach was up to $70 \times$ faster than the simulator. As a result, it was significantly faster to use the analytic approach for the reliability assessment. To obtain an idea of the reliability of NoCs with large number of cores, we determined the fault-resilience using the analytic model for a network of 256 cores, for both mesh and hex topology. The results shown in Fig.8, show that for $15\%$ faulty routers, the hex NoC with a fault resilience of 0.877 is $29\%$ more resilient than the mesh NoC.

![Fig. 7. Results of fault resilience vs. percentage faulty routers for Hex NoC in comparison to cycle-accurate simulation.](image)

VII. CONCLUSION AND FUTURE WORK

In this paper, we presented a flexible analytic approach to assess the reliability of fault-tolerant routing algorithms for NoC based on matrix algebra of the channel dependency matrix. The approach is very general and can be adapted to model any fault-tolerant routing algorithm. The high accuracy of the model was verified by comparison to cycle-accurate simulation results. We investigated two different topologies-the grid mesh and the hexagonal NoC with the negative-first routing algorithm to test the accuracy of the analytic model. The model was able to estimate the fault-tolerance of the network at different fault ratios with an average estimation error of only about $1\%$. As the model is significantly faster than the cycle-accurate simulator, we are able to estimate the fault-resilience on an average $70 \times$ faster allowing us to investigate NoCs of bigger sizes.

An extension of the model to a more stochastic one, based on probabilities of link and router failures is planned, to achieve even higher flexibility. This would allow us to obtain the steady-state link usage probabilities which would be very useful to estimate the network mean latencies using queuing theory based models such as given by authors in [17].

REFERENCES


