Abstract—Increasing demand for higher data rate in mobile communications has recently attracted interest in high frequency communications above 10 GHz, known as millimeter wave (mmW) communications. Challenged by the high path loss, the use of antenna arrays are mandatory. The resulting directional deafness has to be met by effective beam alignment algorithms. In order to keep costs and complexity in acceptable limits, it is likely that first devices for mobile communication will feature analog beam forming with one RF chain. We address this scenario, by introducing an efficient robust beam alignment algorithm and show its performance by simulation.

I. INTRODUCTION

The increasing demand for data rate for mobile communications is pushing traditional cellular systems to its limits. Improving spectral efficiency by using high order modulation and low-out-of-band radiation transmission schemes are only partially able to meet this demand, as the bandwidth in traditional cellular frequencies is limited. Millimeter wave (mmW) communication is believed to meet the throughput demands by the use of large spans of available bandwidth. The ten time increase of the carrier frequency compared to traditional bands causes several challenges for communications. First, the free path loss is higher due to the smaller wavelength, making it necessary to use directional communication and different protocols [1]. Second, radio equipment is hard to build and subject of several impairments and high power loss [2]. As a consequence, it is widely accepted, that the relation of the number of radio frequency (RF) chains to the number of radiating antenna elements has to be reduced compared to the digital beam forming approach commonly used in traditional bands [3], [4]. Different requirements lead to different trade-offs between complexity and costs on the one and flexibility on the other side and different system designs for a variety of use cases. The system designs are ranging from analog beam forming with one RF chain and a fixed set of beam patterns to the hybrid beam forming (HBF) approach with several RF chains and variable phase shifters at each antenna element [5]. Common challenge of all systems in the mmW regime is the need to first establish communication in the presence of high pathloss and second exploiting the channel in an optimal way for maximum throughput. This problem is commonly referred to as (spatial) channel estimation and beam forming. For systems with one RF chain, the problem is called beam alignment for a static scenario and beam tracking for a mobile scenario. Several solutions for these problems exist in literature, where we focus on the beam alignment problem for low complexity solutions with on RF chain. The first category of algorithms, the beam sweeping techniques, solve the alignment by measuring the necessary beam combination to get complete knowledge of the angle of departure (AoD) and angle of arrival (AoA). Beside the most simple exhaustive search, the search can also be done hierarchically [6], leveraging wide beams in the beginning. This method was adapted in standards IEEE 805.15.ac [7] and IEEE 802.11ad [8] for static indoor communication. As the use of wide beams urges the need of large spreading factors, search time is still large compared to other approaches. The second category uses direct AoD and AoA estimation techniques specifically for HBF. These algorithms exploit the sparse structure of the channel by using compressed sensing (CS) techniques [9]–[12]. However, these approaches rely on the exact knowledge of the specific beam patterns and seem to be sensitive on inaccurate knowledge of the antenna weights. Common for this type of algorithm is their need for complex system like HBF and the need for high spreading factors or exhaustive search like channel probing at the begin of the estimation. The third algorithm category use partial indirect channel knowledge. This category of algorithms combine the use properties of the unknown objective functions (e.g. sparsity) with stochastic exploration of the interesting areas of the objective function. Based on mathematical concepts like black-box objective, the algorithms combine the benefit of less probing with avoiding wider beams [13], [14] and [15]. One drawback of the algorithms is the high complexity which prevents the implementation into real systems. The aim of this paper is to present a practical and robust beam alignment algorithm of this category with highly reduced complexity and proving its applicability and superiority of the algorithm by simulation. The paper is structured as follows: Section II defines the system model and assumptions, Section III describes the algorithm in theory and Section IV presents a way for proper parametrization. Section V presents the results of simulation to show the superiority of the algorithm, before Section VI gives a conclusion.

II. SYSTEM MODEL

Given a transmitter and receiver with one RF chain each, the receive signal can be expressed as [4]

\[
y = \sqrt{P_t} w_l^H H k s + w_l^H n
\]  

(1)
where $s$ with $|s| = 1$ is the transmit symbol. $\sqrt{P_t}$ is the transmit power, $f_k \in \mathbb{C}^{N_t \times 1}$ describes the complex phase weighting vector with constant magnitude entries of the transmit antenna, $w_l \in \mathbb{C}^{N_t \times 1}$ the complex phase weighting vector of the receive antenna, $n \in \mathbb{C}^{N_t \times 1}$ the noise vector. The beams for are chosen out of a codebook $\Phi = \{f_k, k = 0, ..., K - 1\}$ of cardinality $K$ for the transmitter and $\Psi = \{w_l, l = 0, ..., L - 1\}$ of cardinality $L$ at the receiver. The codebooks are assumed to be over-complete. The patterns of the codebooks are equidistant distributed in angular domain i.e. $\Theta_T = \{\theta_{T,1} + k \Delta \theta_T, k = 0, ..., K - 1\}$ for the transmit beam, a similar expression for receive codebook $\Phi_R$. The channel is given by [16], [17]

$$H = \sqrt{\frac{N_t N_r}{\rho M_p}} \sum_{m=1}^{M_p} \alpha_m a_R(\Theta_T, m) a_T^H(\Phi_R, m),$$

where $M_p$ is the number of resolvable paths and $\alpha_m \in \mathcal{CN}(0, 1)$ is the i.i.d. complex normal distributed and amplitude of each path. The pathloss $\rho/\mathrm{dB} = 20 \log(4\pi/\lambda) + 10 n_{pl} \log(D)$ with distance $D$ and $n_{pl} = 2.1$ is taken from [1]. The array propagation vector at the transmitter can be defined as

$$a_T(\Theta_D) = \frac{1}{\sqrt{N_t}} \left[ 1, e^{j 2\pi d_s \sin(\Theta_D)}, ..., e^{j 2\pi (N_t-1) d_s \sin(\Theta_D)} \right]^T,$$

with a similar formulation for $a_R$ where $\lambda$ is the wavelength. In the following, we choose the spatial distance between the antenna elements to be $d_s = \frac{\lambda}{2}$ in order to avoid grating lobes. The angles of departure $\Theta_D$ and arrival $\Theta_A$ are uniformly distributed in the range of $\Theta_D, \Theta_A \in (-\pi/3, \pi/3)$. We assume the simple notation is sufficient, as the use of single carrier (SC) with null CP and channel equalization is able to cope with the frequency selective nature of the channel.

### III. Beam Alignment Algorithm

As we only use one RF chain, maximizing the data rate is equivalent to maximizing the received power. The beam alignment problem is therefore defined as finding the entries $k, l$ of the codebook $\hat{\Phi}$, $\hat{\Psi}$, which maximize the receive power

$$J(k, l) = \arg \max_{k, l \in \mathcal{F}} |y(k, l)|^2.$$

The objective function $J(k, l)$ (Fig. 1) is not concave in general, as the combination of the transmit and receive beam patterns in different directions exploits the $M_p$ paths of the channel, each resulting in a maximum of the discrete objective function. Sidelobes have an additional effect on the functions shape. The exhaustive search algorithm solves the problem by evaluation of every possible beam pair combination of the codebooks $\hat{\Phi}$ and $\hat{\Psi}$, i.e. full exploration of function $J(k, l)$. The gradient based beam alignment algorithm in contrast is split in two phases. While the aim of the initial alignment is to point to the area of the objective function where it can be regarded concave in the region of the global maximum by using a minimal set of evaluations, the goal of the fine alignment phase is to enhance the receive power stepwise by using different beam combinations, i.e. finding the global maximum of function $J(k, l)$. The algorithm is described in detail below.

#### A. Initial Alignment

The initial search is applied by probing all beam pair combination of the small codebooks $\Phi_{init}$ and $\Psi_{init}$ with cardinality $K_{init} < K$ and $L_{init} < L$. The entries of the codebooks are part of the full codebook and indexing can be translated (i.e. $k_{init} = ak$) and should be complete in the sense, that they should provide a coarse insight to the shape of the cost function and point to an area of interest. The exact requirements for the initial codebooks are formulated below. The result of the initial search at iteration $n = 0$ is given by

$$J(k_0, l_0) = \arg \max_{k_0, l_0 \in \mathcal{F}, \Psi_{init}} J(k, l).$$

#### B. Fine Alignment

For the fine alignment procedure, the full codebooks $\hat{\Phi}$ and $\hat{\Psi}$ are used. The goal of the fine alignment is to find the optimum beam pair of these codebooks, which gives the maximum receive power. Given the result of the initial search, we can use a gradient based approach to achieve the goal. The step is repeated for several iterations $n$ until the abortion criterion is reached. Following the current beam pair combination at iteration $n$, adjacent beam combinations $k_n \pm 1$ and $l_n \pm 1$ are evaluated. Thus the approximation of the instant gradient is given by

$$\nabla J(k_n, l_n) = \frac{\nabla J_T(k_n, l_n)}{\nabla J_R(k_n, l_n)} \approx \frac{J(k_n + 1, l_n) - J(k_n - 1, l_n)}{2\Delta \Theta_T} \frac{J(k_n, l_n + 1) - J(k_n, l_n - 1)}{2\Delta \Theta_R}.$$

The objective function $J(k, l)$ in general is a function of the mainlobe directions $\Theta_T$ and $\Theta_R$, while the denominator of the derivation is dependent on the design of the codebook.

As the codebook is equally spaced in angular domain and
Algorithm 1: Gradient based beam-alignment

<table>
<thead>
<tr>
<th>Gradient based beam-alignment</th>
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</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
</tr>
<tr>
<td><strong>Initial Alignment</strong></td>
</tr>
<tr>
<td>(i) evaluate all beam combinations of codebooks</td>
</tr>
<tr>
<td>( k \in \mathbf{F}<em>{init}, l \in \mathbf{W}</em>{init} )</td>
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<tr>
<td>(ii) ( (k_0, l_0) = \text{argmax} \ J(k, l) )</td>
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<tr>
<td>( k \in \mathbf{F}<em>{init} ) (w/( \mathbf{W}</em>{init} ))</td>
</tr>
<tr>
<td><strong>Fine Alignment</strong> for iteration ( n )</td>
</tr>
<tr>
<td>(i) evaluate adjacent beam combinations</td>
</tr>
<tr>
<td>{( J(k_n - 1, l_n), J(k_n + 1, l_n), J(k_n, l_n - 1), J(k_n, l_n + 1) } )</td>
</tr>
<tr>
<td>(ii) calculate gradient ( \nabla J(k_n, l_n) )</td>
</tr>
<tr>
<td>(iii) find step size parameter ( \mu_{n+1} )</td>
</tr>
<tr>
<td>(iv) [ \begin{bmatrix} \theta_{n+1} \ \phi_{n+1} \end{bmatrix} = \begin{bmatrix} \theta_n \ \phi_n \end{bmatrix} + \mu_n \nabla J(k_n, l_n) - \frac{\theta_n}{\phi_n}, ]</td>
</tr>
<tr>
<td>(v) stop if ( J(k_n, l_n) &gt; J(k_n \pm 1, l_n \pm 1) ) i.e. ( i, j \in {-1, 1} )</td>
</tr>
<tr>
<td><strong>Output:</strong> beam combination ( k_{final}, l_{final} )</td>
</tr>
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</table>

where \( \mu_n \) is the step size for iteration \( n \). The adjacent beam pair combinations \( k_0 \pm 1, l_0 \pm 1, k_0, l_0 \pm 1 \) are now evaluated and the fine alignment step is repeated. The algorithm is stopped if the maximum is bracketed between two smaller values of the cost function, i.e.

\[ J(k_n, l_n) > J(k_n \pm 1, l_n \pm 1). \] (8)

A complete description of the algorithm is given in Table 1. Figures 2 give an example for the algorithm for a channel.

**IV. DEFINITION OF THE INITIAL CODEBOOK**

In this section, we focus on the problem how to define the codebook for the initial search \( \mathbf{F}_{init} \) and \( \mathbf{W}_{init} \). As the beam width can not change, the entries of the initial have to be part of the full codebook. For this reason, we define the codebooks by taking every \( d_T^{th} \) and \( d_R^{th} \) entry of the full codebook, i.e. \( \mathbf{F}_{init} = \{ f_k, k = 0, d_T, 2d_T, \ldots, K - 1 \} \) and \( \mathbf{W}_{init} = \{ w_l, l = 0, d_R, 2d_R, \ldots, L - 1 \} \). Thus the problem is to find the distances \( d_T \) and \( d_R \) respectively. Without loss of generality, we restrict the problem to the receive dimension in the following discussion by omitting the effect of transmit beam forming, meaning the objective function \( J(k', l) \) is dependent only from receive beam index \( l \), by fixing \( k = k' \), \( d_R \) can be found in the same way.

The physical interpretation is that two planar waves with power \( P_1 > P_2 \) approaching the receive array with \( N_R \) antennas. Then, Eq. 1 reduces to \( y = T_R w a_1 \sqrt{2} + T_R w a_2 \), thus the objective function \( J(l) = |y(l)|^2 \) can be obtained by evaluating all entries of the full codebook \( \mathbf{W} \) with angular difference \( \Delta \phi_R \) between mainlobe directions of adjacent beams. From this equation it can be also seen, that in the area, where the effect of one wave will be dominant, the shape of the objective function follows the shape of the beampattern. Note that we assume the full exploration (exhaustive search) of the objective function here to get insight of the typical structure of the function \( J(l) \), while the goal is to shrink the number of needed evaluations in the actual beam alignment algorithm.
to a minimum. We use an over-complete codebook $\mathbf{W}$ with large overlap between adjacent beam patterns. The overlap between adjacent beam patterns can be expressed by using the half power beamwidth (HPBW) of the beam patterns and the angular distance $\Delta \phi$ between adjacent beam patterns. The HPBW is commonly approximated by

$$HPBW = \frac{\pi}{N}. \quad (9)$$

We define the overlap factor (OLF) as a measure of the over-completeness of the codebook as

$$OLF = \frac{HPBW}{\Delta \phi}. \quad (10)$$

Assuming the beam pattern shape to be identical, the cost function gives a sampled version of the beam pattern for each path (Fig. 1). For further investigations, we only focus on main lobes omitting the sidelobes for a moment, and assume the angular difference of the two paths $\phi_{A,1}$ and $\phi_{A,2}$ is large enough, thus that the beam pattern shapes or slopes do not overlap until the noise level. An example for the one dimensional cost function is given in Fig. 3. The maximal receive power exited by wave 1 is given by choosing receive beam pattern $k_{p,1}$ with $J(l_{p,1}) = J_{p,1}$, giving the global maximum of the cost function. The maximal receive power exited by wave 2 is given by choosing receive beam pattern $k_{p,2}$ with $J(l_{p,2}) = J_{p,2}$, resulting in a local maximum. The goal of the beam alignment algorithm is to choose the global maximum by minimizing the number of evaluations of beam patterns. If the result of the initial search gives a beam pair around the global optimum, the slope of the beam pattern will cause the gradient to find the global maximum. If, in contrast the initial search result gives a beam around the local optimum, the slope of the beam pattern will guide the gradient only to the local maximum of the objective function. In order to ensure that the global maximum is found, we have to make sure, the initial search evaluates at least one beam pattern, where the shape of the peak caused by wave 1 is higher than the maximum value of the cost function caused by wave 2. The number of beam patterns in this area of the cost function determines the distance $d_R$ we can allow. As described above, the area of the objective function, which is defined by the influence of wave 1, is given as a sampled version of the beam pattern. The beam pattern for a linear array in power domain is given by

$$g(\phi - \phi_1) = \sin^2 \left( \frac{2\pi \phi}{2} \sin \phi - \sin \phi_1 \right) \quad (11)$$

$$g(\phi - \phi_1) \approx A \cdot e^{-\frac{(\phi - \phi_1)^2}{2}}. \quad (12)$$

where $(a)$ follows an approximation of mainlobe of the beampattern by a Gaussian function. Parameter $A$ can be easily found by investigating the maximum of the pattern function

$$A = \max g_l(\phi) = N^2. \quad (13)$$

![Fig. 4: Approximation of the beam pattern function. The error of the approximation will increase with rising $\phi_0$ as the beamwidth is widened, resulting in a smaller modeled beamwidth. As this is a disadvantage for the approximation can be seen as worst case.](image)

We choose $\sigma = 1/3\phi_0$ because the integral over the range of $-3\sigma...3\sigma$ contains about 99.7% of the area of the Gaussian shaped function.

The condition, that the area of the cost function $J_1 = \{ J(l_{p,1} - l_\Delta), \ldots, J(l_{p,1} + l_\Delta) \}$ has to be larger than the value of the objective function caused by wave 2 $J(l_{p,2})$ leads to the following relation

$$\forall J \in J_1 > J(l_{p,2})$$

$$J(l_{p,1} + l_\Delta) > J(l_{p,2}) \quad (19)$$

$$e^{-\frac{2^2\Delta \phi^2}{2\sigma^2}} > \frac{J(l_{p,2})}{J(l_{p,1})}. \quad (21)$$

with $\phi_\Delta$ is the angular difference between the main lobes and where $(a)$ follows from the fact, that the shape of the maximum of the objective function equals the beam shape and $(b)$ follows from the fact that at least one point of the initial search has to be in $J$, i.e. $2l_\Delta = d_R\phi_\Delta$. After some calculations, by using Eq. 9 and Eq. 10 $d_R$ is given by

$$d_R < \frac{2\sigma}{\Delta \phi} \sqrt{\frac{2 \ln J(l_{p,1})}{J(l_{p,2})}} \quad (22)$$

$$d_R < \frac{\sigma N OLF}{\pi} \sqrt{\frac{8 \ln J(l_{p,1})}{J(l_{p,2})}}. \quad (23)$$

These considerations give a closed form expression of the allowed distance $d_R$ depending on the power of the two planar waves. As this parameter is dependent on the design of the full codebook $\mathbf{W}$, it is no surprise, that it is dependent on the OLF. Note that the number of additional waves with smaller power is independent on our considerations.
A. Effect of Sidelobes

Until now, we did only regard the effect of the mainlobe to objective function \( J \). Sidelobes influence the shape of the function by producing smaller local maxima in addition to a dominant maximum by the main lobes. Therefore, from perspective of the objective function, sidelobes have the same effect as incoming waves with small power. Thus we can model the effect of the sidelobes by a shifted version of the beam pattern with smaller power. For this reason an important model the effect of the sidelobes by a shifted version of the function by producing smaller local maxima in addition to a dominant maximum by the main lobes. Therefore, from objective function \( J \).

Thus the relation between the mainlobe and first side lobe gain is

\[
J(l_{p,2}) = J(l_{p,1}) \frac{1}{\sin(\pi/2 \cdot 3/N_R^2 \cdot N_R^2)}.
\]  

B. Expectation Value

If the relation \( J(l_{p,2}) / J(l_{p,1}) \) becomes smaller the distance parameter \( d_R \) has to be decreased (Eq. 23). If the relation becomes too small, the distance has to be \( d_R = 1 \), meaning the initial search is equaling exhaustive search and no benefit in terms of decreasing the number of evaluations is reached. Nevertheless the probability of finding the global maximum over different channel realizations can be quite high using a larger \( d_R \). To give an understanding of the correct parametrization in these cases, we use the expectation value of the maximum of the objective function over several channel realizations found by the algorithm \( J_{alg} \), compared to the global maximum of the objective function \( J_{max} \). In contrast to the case above we cannot ensure the global maximum is reached, but we can control the probability that the global optimum is found. Assume again a scenario, where two planar waves with power \( P_1 > P_2 \) but \( P_1 \approx P_2 \) are hitting one array with \( N_R \) antennas.

Given a targeted expectation of the relation value over several channel iterations \( \mathbb{E} \left\{ J_{alg} \right\} < 1 \), the goal is to choose the distance parameter \( d_R \) in dependency of the OLF property of the codebook. We again assume no overlap and assume the shape caused by the waves are symmetrical sampled by the beam patterns (Fig. 5). The expectation value can be described using the probability of finding the local or global maximum by

\[
\mathbb{E} \left\{ J_{alg} \right\} = \Pr \left\{ J(l_{p,1}) \right\} J(l_{p,1}) + \Pr \left\{ J(l_{p,2}) \right\} J(l_{p,2})
\]

where \( N_R \) describes the noise level. Finding the global maximum of a beam initial search is located around the slope of global maximum 1 or local maximum 2 respectively, i.e. the power of the receive signal has to higher than the noise. In the following, we name the set of all beam patterns probed for the initial search \( \mathcal{W} \) and the resulting set of cost function values \( J_{init} \).

If the beam pattern which gives the local optimum is in the initial codebook \( l_{p,2} \in \mathcal{W} \), we cannot ensure the global maximum is reached, but we can control the probability that the local optimum \( l_{p,1} \) is part of \( J_{init} \) and is limited by one. The probability, that \( l_{p,2} \in \mathcal{W}_{init} \) is

\[
\Pr \left\{ l_{p,2} \in \mathcal{W}_{init} \right\} = \frac{1}{d}.
\]
where \( m \) is the maximum.

Fig. 5: One dimensional objective function with one global and a local peak. 3 adjacent beams are at the top of the global maximum.

Thus the probability, that the maximum received power of the initial search can be calculated according to Eq. 29 by

\[
m < \frac{\sigma}{\Delta \phi} \sqrt{2 \ln \frac{J_{p,1}}{J_{p,2}} + \frac{n^2 \Delta \phi^2}{\sigma^2}} \quad (36)
\]

\[
m < \frac{\sigma \text{NOLF}}{\pi} \sqrt{2 \ln \frac{J_{p,1}}{J_{p,2}} + \frac{n^2 \pi^2}{N^2 \text{OLF}^2 \sigma^2}} \quad (37)
\]

\[
m < \sqrt{\frac{2\sigma^2 N^2 \text{OLF}^2}{\pi^2} \ln \frac{J_{p,1}}{J_{p,2}} + n^2}. \quad (38)
\]

The expectation value can now be given by

\[
E \{ J_{\text{alg}} \} = \Pr \{ J_{p,1} \} J_{p,1} + \left( 1 - \Pr \{ J_{p,1} \} \right) \Pr \{ J_{\text{alg}} = N \sigma_n^2 \} J_{p,1} + \Pr \{ P = N \sigma_n^2 \} N \sigma_n^2
\]

with

\[
\Pr \{ J_{\text{alg}} = N \sigma_n^2 \} = (1 - \min \frac{2m_{\text{all}} + 1}{d})(1 - \min \frac{2n_{\text{all}} + 1}{d}). \quad (46)
\]

Fig. ?? ?? show the expectation value of the receive power over OLF and different relations of the power \( \frac{J_{p,1}}{J_{p,2}} \). In several investigations, an independence of \( N \) was approved. In contrast, an dependence on \( d \) is existing.
Thus the approximation of the optimal parameter $\mu$ function is falling instead of rising. This situation can be determined if the result of the objective function is assumed to be concave in an area around the maximum value of the initial alignment and thus can be approximated as a parabola. Each iteration we get three adjacent points of the objective function, which can be used to determine the parameters of the underlying parameters. The step size parameter $\mu_n$ can be set, by finding the maximum point of the approximation.

Defining the approximation of the objective function in dimension $l$ by fixing $k = k'$

$$J(k', l_n) = a\phi_{l_n}^2 + b\phi_{l_n} + c ,$$  \hspace{1cm} (47)

parameter $a$, $b$ and $c$ can be calculated by using $J(k_n, l_n - 1)$, $J(k_n, l_n)$ and $J(k_n, l_n + 1)$ using standard linear algebra. The argument for the maximum value of $J(l_n)$ is given by

$$\phi_{l_{\text{max}}} = -\frac{b}{2a}$$ \hspace{1cm} (48)

Thus the approximation of the optimal parameter $\mu_R$ for the transmit dimension $k$ and the total step size parameter $\mu$ is given by

$$\mu_{k+1} = \sqrt{\mu_k^2 + \mu_R^2}.$$ \hspace{1cm} (50)

Fig. 8 shows an example for the usage of this approximation. It is possible, that the global maximum is missed due a too large step size parameter $\mu$, resulting in a bracketing of the maximum two beampair candidates in rising iteration number. This situation can be determined if the result of the objective function is falling instead of rising.

$$J_n(k, l) < J_{n-1}(k, l)$$ \hspace{1cm} (51)

In these situations we decrease the step size parameter

$$\mu_{n+1} = \frac{\mu_n}{2}.$$ \hspace{1cm} (52)

In other situations, parameter $\mu$ might be too small to initiate a beam change. Thus we define the lower bound of the parameter $\mu$ by taking into account the minimum step size needed to change to the adjacent beam pattern

$$\mu_{\text{min}} = \sqrt{\frac{\Delta \phi_R^2 + \Delta \phi_T^2}{\nabla J(k_n, l_n)_T^2 + \nabla J(k_n, l_n)_R^2}}.$$ \hspace{1cm} (53)

It is clear, if the OLF is low or distance is high, the probability is higher to miss the maximum. If the OLF becomes high, the probability to find the maximum is high. If the ratio $\frac{P_{\text{max}}}{P_{\text{max}}}$ is close to one, it does not matter, which of the two maximum is found.

V. SIMULATION RESULTS

We now prove the applicability of the system by simulation. We use the channel model introduced in Eq.2 for a small cell scenario with $M_p = 2$ paths with a complex Gaussian distributed amplitude and independent uniform distributed AoA $\phi_A$ and AoD $\phi_D$ at a center frequency of 73 GHz. The transmitter and receiver are assumed to have a linear array of each $N_T = N_R = 5$ antenna elements. The defined codebook with cardinality $|\mathcal{F}| = |\mathcal{W}| = 25$, which are equidistant distributed in angular domain in the range of $\phi \in [-\pi/3, \pi/3]$ with angular distance of $\Delta \phi_T = \Delta \phi_R = 5^\circ$ has an OLF of $OLF \approx 7$. We have to know, what is the maximum distance $d_R = d_T$ for the initial search for the given OLF. We want to be sure, that the effect of the sidelobes can be always neglected. Based on Eq. 23 and Eq. 28, we can calculate $J(k_n, l_{n+1}) = 0.061$ and thus $d < 7$. For the two paths we specify our system as follows. We want to allow a relation of $\frac{P_{\text{max}}}{P_{\text{max}}}$ to be $\frac{1}{2}$, and an expectation value of $E[\left(\frac{J_{\text{algo}}}{J_{\text{max}}}\right)] = 0.97$ for the given OLF. Therefore, we choose $d = 4$ as a larger distance result in a smaller expectation value (Fig. 7c).

We compared the performance in three ways. First, the relation $J_{gr}/J_{es}$ compares the receive power given by the gradient search and exhaustive search for the same channel realization. Fig. 9 shows, that the proposed method achieves almost the same receive power as exhaustive search with raising SNR, as a too large noise floor destroys the smooth nature of the objective function making the gradient information useless. The second performance criterion is the relation of beam pair evaluation, which is needed for algorithm termination $\text{ev}_{gr}/\text{ev}_{es}$ (Fig. 10). It shows, that the gradient needs at most $7.2\%$ of the probing effort of the full exhaustive search. The third performance criterion, achievable normalized rate $[15]$ is depicted in Fig. 11. Although the gradient algorithm achieves significant less receive power as the exhaustive search for very bad channel conditions, it does not reflect in the achievable rate, as the channel is anyway too bad to be used for communications for the given system. In summary the results show, that the proposed gradient based algorithms
achieves almost the same performance in receive power and rate for values of the SNR $> -20$ dB, where communication is meaningful. The disadvantage of little smaller receive power is compensated by the advantage of only using about 7% of the probing compared to the exhaustive search algorithm. However this comes at the cost of additional feedback, as the number of iterations between transmitter and receiver are rising.

VI. CONCLUSION

This paper presents an experimental evaluation of a novel steepest-descent like approach for beam alignment for low cost mmW systems. The algorithm applies initial search over a quasi-orthogonal codebook with refinement using approximate gradient using an over-complete codebook. After proper parametrization, simulations show almost the same achievable rate as exhaustive search by using only up to 8% of the probing at the cost of some feedback.

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