SNR-Aware Power Allocation Scheme for Lossy-Forward Relaying Systems

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***Abstract*—In a recent work, we proposed a simple and efficient power allocation scheme for a dual-hop communication system with multiple lossy-forward relays. In this work, we propose a less simple, yet more efficient and comprehensive power allocation scheme for the referred system. Unlike our previous approach, which was based on an asymptotic reasoning at high signal-to-noise ratio, the new one optimizes the allocation policy according to any particular amount of total transmit power. The more constrained that amount of power, the more advantageous the new scheme.**

***Index Terms*—CEO problem, lossy-forward relaying, power allocation, Rayleigh fading.**

I. Introduction

In some important emerging applications, sources and destinations shall be designed to communicate through multiple, simultaneous, parallel routes, each of which is likely to be composed of multiple hops experiencing unusually adverse channel conditions. Such applications include, for instance, links-on-the-fly emergency networks operating over areas devastated by natural disasters and fifth-generation vehicular networks operating under severe, highly dynamic propagation environments [1]. In those cases, standard relaying protocols like conventional decode-and-forward (DF) may prove inappropriate, as retransmissions can be invoked too often in view of the adverse channel conditions[1]. This could render unfeasible the communication process as a whole.

A more fundamental aspect that discourages the use of conventional DF relaying in the referred application scenarios is that, instead of having the relays discard an erroneous message and invoke a retransmission, it is indeed advantageous to have them forward such a message to the destination [2], [3]. This finding gives rise to the so-called lossy-forward (LF) relaying protocol, a.k.a. DF relaying with intra-link errors. In that scheme, the relay messages are always forwarded to the destination, regardless of any errors they may contain. The main idea is that even erroneous relay messages can be jointly decoded at the destination, yielding a source message estimate that is more accurate than each relay message alone.

In LF relaying systems, the destination aims for an optimal retrieval of the source message, given the erroneous replicas from the relays. However, as these replicas are corrupted, the optimum retrieval cannot be error-free, whatever the coding scheme. In information theory, finding the ultimate performance limits for such a coding scheme is known as the chief executive officer (CEO) problem [4]. The problem is still open in its general form, i.e., it is so far unknown what is the minimum set of rates (a.k.a. the rate-distortion region) at which the relay encoders can communicate with the destination while still conveying enough information to satisfy a given distortion constraint on the reconstruction of the source message.

From a more practical viewpoint, one problem of interest is how to allocate a given amount of transmit power among a group of LF relays in a way that minimizes the end-to-end probability of error. As mentioned above, no general exact rate-distortion region for the CEO problem exists yet. On the other hand, a relaxed information-theoretical framework can be applied to the power allocation problem at hand. In [1], for example, based on a suitably modified version of the Slepian-Wolf rate region [5] along with an asymptotic analysis at high signal-to-noise ratio (SNR), we proposed a simple and efficient power allocation scheme for an LF system containing an arbitrary number of relays. In this work, based on a rather distinct approach — maximizing the mutual information between the input and the output of a simplified, end-to-end, equivalent channel — we propose a new power allocation scheme for that same LF system. Our new scheme proves mathematically less simple, yet more efficient and comprehensive than that we proposed in [1].

In the remainder of the text, $\Pr[\cdot]$ denotes probability; $f_{(\cdot)}$, probability density function; $p_{(\cdot)}$, probability mass function; $\mathbb{1}\{\cdot\}$, the indicator function, which yields 1 if its argument is true, and 0 otherwise; $h(\cdot)$, differential entropy; $I(\cdot;\cdot)$, mutual information; $\mathcal{B}$, the binary set $\{-1,+1\}$; $\mathcal{N}$, the set $\{1,\ldots,N\}$; $\{A_i\}_{\mathcal{S}}$, an indexed series, e.g., $\{A_i\}_{\{1,2,3\}} = \{A_1, A_2, A_3\}$; and "$\sim$", "asymptotically equal to."

II. System Model

As depicted in Fig. 1, we consider that one source communicates with one destination through an arbitrary number N of LF relays. The source generates an independent, identically

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[1]In conventional DF relaying, whenever a relay detects an erroneous message, the message is discarded and a retransmission is invoked.

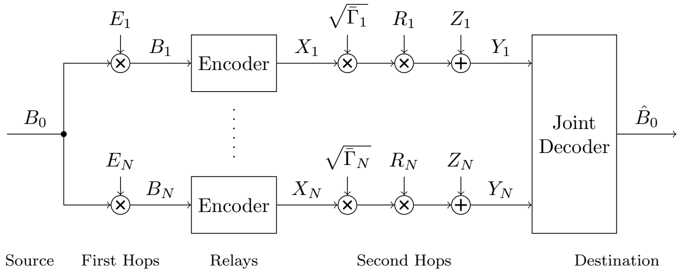

Fig. 1: Lossy-forward relaying system.

distributed (i.i.d.), uniform binary sequence[2] $B_0 \in \{\pm 1\}$, i.e., $\Pr[B_0 = -1] = \Pr[B_0 = 1] = 1/2$. The sequence is broadcast through independent, memoryless, binary symmetric channels with crossover probabilities $0 < p_i \leq 1/2$, $i \in \mathcal{N}$. So the retrieved sequence at the ith relay can be written as $B_i = B_0 \times E_i$, with $E_i \in \{\pm 1\}$ being an i.i.d. binary error sequence associated with the ith first hop, according to $\Pr[E_i = -1] = p_i$ and $\Pr[E_i = 1] = 1 - p_i$. The relay sequences are re-encoded and forwarded to the destination, regardless of any errors they may contain[3]. The second hops undergo dissimilar Rayleigh fading and additive white Gaussian noise. For simplicity, the transmitted symbols X_i, the fading coefficient R_i, and the noise term Z_i at the ith second hop are assumed to have unity mean powers, with the received average SNR being encapsulated in a single deterministic gain, $\bar{\Gamma}_i$. This gain is expressed as $\bar{\Gamma}_i = \alpha_i (P_{\mathrm{T}}/N_0) d_i^{-\eta}$, where P_T is the total transmit power shared by the relays; $0 \leq \alpha_i \leq 1$, a power allocation coefficient, with $\sum_{i=1}^N \alpha_i = 1$, so that $\alpha_i P_T$ is the transmit power at the ith relay; N_0, the mean noise power; d_i, the distance between the ith relay and the destination; and η, the path-loss exponent. Finally, by jointly decoding the received replicas $\{Y_i\}_{\mathcal{N}}$, the destination estimates the source sequence as $\hat{B}_0$.

One of the assumptions above deserve further justification. In potential application scenarios, the source sequence is likely to be transmitted through multiple routes of multiple hops each. And all of those hops — not only the last ones — are expected to undergo some type of fading. In fact, due to this very fading, along with additive noise, the relay at the ith last hop retrieves the source sequence according to a certain crossover probability, p_i. It turns out that these values of $\{p_i\}_{\mathcal{N}}$ are all we need to devise an efficient power allocation scheme for the last-hop relays. This is why we adopted a simple amalgamated model — the binary symmetric channel — to jointly describe the multiple hops up to the last one.

III. Problem Statement

We want to find a set of power allocation coefficients $\{\alpha_i^*\}_{\mathcal{N}}$ that minimize the end-to-end probability of error for the LF relaying system. In other words, we want to solve the following optimization problem:

$$\begin{aligned} \{\alpha_i^*\}_{\mathcal{N}} = &\arg\min_{\{\alpha_i\}_{\mathcal{N}}} \quad \Pr[\hat{B}_0 \neq B_0] \\ &\text{subject to} \quad 0 \leq \alpha_i \leq 1 \text{ and } \sum\nolimits_{i=1}^N \alpha_i = 1. \end{aligned} \tag{1}$$

The ordinary approach for tackling the above problem is to express $\Pr[\hat{B}_0 \neq B_0]$ in terms of $\{\alpha_i\}_{\mathcal{N}}$ and then to search for its minimum via analytical or numerical methods. This approach is rather convoluted, however. First, the mathematical formulation involved would change according to the particular coding scheme used at the relays and to the particular joint decoding scheme used at the destination. Second, and most importantly, given any such combination of coding and decoding schemes, expressing $\Pr[\hat{B}_0 \neq B_0]$ in terms of $\{\alpha_i\}_{\mathcal{N}}$ may be extremely difficult or even impracticable, to say nothing of the subsequent optimization.

In view of that intricacy, alternative approximate solutions prove appealing. In Section IV, we summarize a simple, yet effective existing solution; in Section V, we propose a less simple, yet more efficient and comprehensive one. Both solutions are based on information-theoretical metrics somehow connected to $\Pr[\hat{B}_0 \neq B_0]$, the original objective function.

IV. Existing Solution Revisited

The relay sequences $\{B_i\}_{\mathcal{N}}$ can be regarded as mutually correlated sources of information. Owing to that, in [1] we capitalized on a modified version of the Slepian-Wolf rate region in order to design an efficient power allocation scheme for an LF system containing an arbitrary number of relays. The modification of the theorem's scope was required because the destination is ultimately not interested in recovering the set of noisy relay sequences, but the source sequence instead, namely B_0. By using this reasoning to define a somewhat relaxed rate region, and maximizing the high-SNR asymptotic probability of meeting that region, we eventually arrived at a remarkably simple power allocation policy, as follows:

$$\alpha_i^\star = \frac{\sqrt{C_i d_i^\eta}}{\sum_{j=1}^N \sqrt{C_j d_j^\eta}}, \; i \in \mathcal{N}, \tag{2}$$

where $C_i \triangleq 2^{R_c I(B_i; B_0 | \{B_j\}_{j \neq i})} - 1$, with R_c representing the spectral efficiency associated with the modulation and coding schemes used at the relays, and with each mutual information $I(B_i; B_0 | \{B_j\}_{j \neq i})$ being provided in [1] as a function of the crossover probabilities $\{p_i\}_{\mathcal{N}}$. Since the allocation policy in (2) is based on the asymptotic optimization of a performance metric, we called it asymptotically optimal power allocation (AOPA). That allocation depends ultimately on four classes of system parameters: the crossover probabilities at the first hops, $\{p_i\}_{\mathcal{N}}$; the relay-destination distances, $\{d_i\}_{\mathcal{N}}$; the spectral efficiency of the relaying transmission, R_c; and the path-loss exponent, η. Despite its mathematical simplicity, the AOPA scheme proves quite effective, outperforming equal-power allocation (EPA) over a wide range of SNR, as shown and discussed in [1].

At first glance, it looks rather surprising that the AOPA scheme is able to achieve such a good performance. On the

[2] To alleviate the notation, temporal indices are omitted.

[3] This is the only difference between lossy-forward relaying and conventional decode-and-forward relaying. In the latter, any erroneous message is discarded by the relays.

one hand, this allocation scheme turns out to diminish the end-to-end bit-error rate (BER) at low to medium SNR, as intended. On the other, it has been designed to minimize a certain relaxed-sense outage probability at the high-SNR regime. Interestingly, at the high-SNR regime, trying to minimize the BER directly would be pointless for LF relaying systems. In those systems, as the SNR increases, whatever allocation scheme[4] eventually merges into a same fixed level of BER — an equivalent noise floor governed by the crossover probabilities at the first hops. This can be seen from the BER curves presented in [1]. All in all, the information-theoretical framework built for the AOPA scheme is somehow able to mirror, even in its asymptotic simple form, how the power allocation among the relays mainly affects the BER.

Despite the good performance achieved by the AOPA scheme, the above reasoning suggests there might be room for improvement. Indeed, by testing exhaustive allocations, one can verify that the room increases when the crossover probabilities at the first hops are more dissimilar, as well as when the relays are more unequally apart from the destination. In the next section, we introduce a new power allocation paradigm that achieves such an improvement.

V. Proposed Solution

In essence, we propose to choose a power allocation that maximizes the mutual information between the input and the output of an end-to-end equivalent channel for the investigated system. Although this is a basic design principle in information theory, applying it to our system model proves rather tricky, as discussed next.

The difficulty mainly arises because of the relays' encoders taking part of the end-to-end equivalent channel, as depicted in Fig. 1. On the one hand, a given set of relays' encoders add no randomness to the analysis, because they perform a deterministic mapping between, say, m data bits B_i, to, say, n coded symbols X_i. On the other, finding an exact single-letter expression for the target mutual information between the end-to-end equivalent channel input B_0 and output $\{Y_i\}_{\mathcal{N}}$ proves challenging, because the encoding function is a block-wise operation (rather involved, usually).

Alternatively, we propose an approximate solution, as follows. Of course, the relays' encoders are essential to the good functioning of the communication system. The better designed the encoders, the smaller the BER. However, since the encoders perform a deterministic processing of the input data, we conjecture[5] that, whatever the encoders, the power allocation that minimizes the BER should barely if not change — though the minimum BER value may change considerably. By pushing that reasoning to the limit, we can drop the relays' encoders altogether[6]. This is done in Fig. 2.

[4]This is true provided that all relays get a nonzero percentage of the total transmit power.

[5]Most importantly, our conjecture is confirmed by the numerical examples to be presented in the next section.

[6]This is equivalent to using uncoded binary phase-shift keying at the relays.

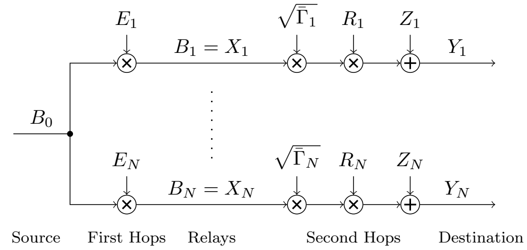

Fig. 2: Constrained end-to-end equivalent channel.

From Fig. 2, the constrained end-to-end equivalent channel leads to the following input-output relationship:

$$\begin{aligned} Y_1 &= \sqrt{\alpha_1 (P_\mathrm{T}/N_0) d_1^{-\eta}} \cdot B_0 \cdot E_1 \cdot R_1 + Z_1 \\ Y_2 &= \sqrt{\alpha_2 (P_\mathrm{T}/N_0) d_2^{-\eta}} \cdot B_0 \cdot E_2 \cdot R_2 + Z_2 \\ &\vdots \\ Y_N &= \sqrt{\alpha_N (P_\mathrm{T}/N_0) d_N^{-\eta}} \cdot B_0 \cdot E_N \cdot R_N + Z_N . \end{aligned} \tag{3}$$

Accordingly, our proposal of maximizing the mutual information between the channel input and the channel output can be written as

$$\begin{aligned} \{\alpha_i^*\}_{\mathcal{N}} = &\arg \max_{\{\alpha_i\}_{\mathcal{N}}} \quad I(B_0; \{Y_i\}_{\mathcal{N}}) \\ &\text{subject to} \quad 0 \le \alpha_i \le 1 \text{ and } \sum\nolimits_{i=1}^{N} \alpha_i = 1. \end{aligned} \tag{4}$$

To solve (4), we need to obtain $I(B_0; \{Y_i\}_{\mathcal{N}})$ in terms of $\{\alpha_i\}_{\mathcal{N}}$. This can be attained from

$$I(B_0; \{Y_i\}_{\mathcal{N}}) = h(\{Y_i\}_{\mathcal{N}}) - h(\{Y_i\}_{\mathcal{N}} | B_0), \tag{5}$$

with $h(\{Y_i\}_{\mathcal{N}})$ being formulated as a function of the joint density of $\{Y_i\}_{\mathcal{N}}$ as

$$\begin{aligned} h(\{Y_i\}_{\mathcal{N}}) = -\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} & f_{\{Y_i\}_{\mathcal{N}}}(\{y_i\}_{\mathcal{N}}) \times \\ & \log_2 \left(f_{\{Y_i\}_{\mathcal{N}}}(\{y_i\}_{\mathcal{N}}) \right) dy_1 \cdots dy_N , \end{aligned} \tag{6}$$

and $h(\{Y_i\}_{\mathcal{N}} | B_0)$ being formulated as a function of the conditional joint density of $\{Y_i\}_{\mathcal{N}}$, given B_0, as

$$\begin{aligned} h(\{Y_i\}_{\mathcal{N}} | B_0) &\overset{\text{(a)}}{=} h(\{Y_i\}_{\mathcal{N}} | B_0 = -1) = \\ & - \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_{\{Y_i\}_{\mathcal{N}} | B_0}(\{y_i\}_{\mathcal{N}} | -1) \times \\ & \log_2 \left(f_{\{Y_i\}_{\mathcal{N}} | B_0}(\{y_i\}_{\mathcal{N}} | -1) \right) dy_1 \cdots dy_N , \end{aligned} \tag{7}$$

where, in step (a), we have exploited the symmetry of the binary channel models used for the first hops.

It remains to find $f_{\{Y_i\}_{\mathcal{N}}}(\cdots)$ and $f_{\{Y_i\}_{\mathcal{N}}|B_0}(\cdots|\cdot)$, as follows. Note from (3) that, conditioned on B_0 and $\{E_i\}_{\mathcal{N}}$, each channel output Y_i is a sum of a scaled Rayleigh random variable and a standard Gaussian random variable. The density function of such a sum, say, $Y \triangleq kR + Z$ — k, R, and Z being an arbitrary constant, a unity-power Rayleigh random variable, and a zero-mean, unity-variance Gaussian random variable,

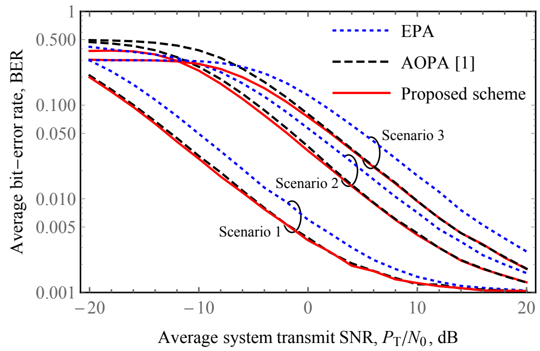

Fig. 3: Bit-error rates for two relays.

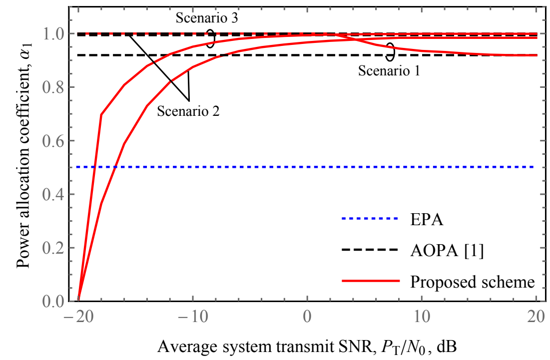

Fig. 4: Power allocation coefficients for two relays.

respectively — can be derived by convolving the individual density functions. After algebraic simplifications, we obtain

$$f_Y(y;k) = e^{-\frac{y^2}{2}} \times \frac{\left[\sqrt{2\left(k^2+2\right)} + \sqrt{\pi} k y e^{\frac{k^2 y^2}{2\left(k^2+2\right)}} \left(1 + \operatorname{erf}\left(\frac{ky}{\sqrt{2(k^2+2)}}\right)\right)\right]}{\sqrt{\pi\left(k^2+2\right)^3}}. \tag{8}$$

Finally, taking into account all (binary) combinations of B_0 and $\{E_i\}_{\mathcal{N}}$, we arrive at

$$f_{\{Y_i\}_{\mathcal{N}}}(\{y_i\}_{\mathcal{N}}) = \sum_{b_0, \{e_i\}_{\mathcal{N}} \in \mathcal{B}^{N+1}} \frac{1}{2} \prod_{i=1}^{N} p_{E_i}(e_i) f_Y\left(y_i; \sqrt{\alpha_i (P_T/N_0) d_i^{-\eta}} b_0 \cdot e_i\right) \tag{9}$$

$$f_{\{Y_i\}_{\mathcal{N}}|B_0}(\{y_i\}_{\mathcal{N}}|b_0) = \sum_{\{e_i\}_{\mathcal{N}} \in \mathcal{B}^{N}} \prod_{i=1}^{N} p_{E_i}(e_i) f_Y\left(y_i; \sqrt{\alpha_i (P_T/N_0) d_i^{-\eta}} b_0 \cdot e_i\right), \tag{10}$$

where $p_{E_i}(e) = p_i \mathbb{1}\{e=-1\} + (1-p_i)\mathbb{1}\{e=1\}$.

In short, the power allocation coefficients are obtained by substituting (9) and (10) into (6) and (7), and these into (5), so as to compose the target function in (4), which can be then maximized via standard numerical optimization routines available in computing softwares such as Matlab and Mathematica. Although this approach may prove impracticable for large values of N due to the multidimensional numerical integration required in (6) and (7), it proves fully tractable for small values of N, say, up to five or six — most likely to be adopted in practice.

VI. Numerical Results

Fig. 3 compares BER simulations[7] of the EPA and AOPA schemes with those of the new power allocation scheme proposed here. For illustration purposes, we assume a two-relay system with $p_1 = 0.001$, $p_2 = 0.3$, $R_c = 1/2$, and $\eta = 4$. As for the relay-destination distances, three cases are considered: $d_1 = 1/3$ and $d_2 = 2/3$ (Scenario 1); $d_1 = 2/3$ and $d_2 = 1/3$ (Scenario 2); and $d_1 = 9/10$ and $d_2 = 1/10$ (Scenario 3). Note how the proposed scheme outperforms the AOPA scheme in all the cases and over the entire range of SNR, markedly at low to medium SNR, as expected. The advantage is more appreciable when the least reliable relay (the one with the highest crossover probability) is closest to the destination, as in Scenario 3, and is less appreciable otherwise, as in Scenario 1.

Fig. 4 shows the corresponding power allocation coefficients. Note how the AOPA scheme prescribes a fixed power allocation for a given scenario, whereas the proposed scheme adapts their allocation coefficients along the range of SNR. At occasions, those recommendations greatly disagree. In Scenario 2, for instance, the AOPA scheme prescribes virtually no power to the second relay at a transmit SNR of -20 dB, whereas the proposed scheme prescribes right the opposite. In Scenario 3, for that same level of transmit SNR, the AOPA scheme barely changes its recommendation, whereas the proposed scheme suggests turning off the first relay altogether. Those are representative scenarios in which the AOPA scheme is quite inaccurate at low SNR, and the proposed scheme manages to overcome this drawback.

VII. Conclusions

We introduced a new information-theoretical framework for allocating power to lossy-forward relays. The new framework optimizes the allocation over the entire range of transmit power, thereby outperforming — although being less simple than — a previous approach based on asymptotic arguments at high signal-to-noise ratio.

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