Combined Filter for Sample Rate Conversion, Matched Filtering, and Symbol Synchronization in Software Radio Terminals
Matthias Henker and Gerhard Fettweis, Dresden University of Technology, Mannesmann Mobilfunk Chair for Mobile Communications Systems, D-01062 Dresden, Germany

Abstract
The idea of software radio (SWR) implies the capability of changing the air-interface just by downloading the respective software. Since analog components (e.g., for pre-filtering and digitization) are difficult to parameterize these tasks have to be moved to the digital domain, or have to be done in a standard independent way. In such receivers the task of sample rate conversion (SRC) is essential and has to be performed in an adaptable manner. As a process of resampling SRC requires anti-aliasing filtering. Once a parameterizable platform for filtering has been designed it is sensible not only to realize anti-aliasing filtering (for SRC), but also other filtering tasks on this platform e.g., for channelization, matched filtering, or symbol synchronization. This enables to reduce complexity. By using a novel implementation of filters it is shown that the complexity can be reduced drastically, and that the performance can be made independent from the chosen rate change factor.

1 Introduction
Shifting tasks from the analog to the digital domain is one of the main issues of software radio in order to enable easier reconfiguration. Therefore, the received signal must be digitized very close to the antenna. Typically the digitization process is neither synchronized to the data symbols / chips, nor to the sample rate of the selected standard of operation. A consequence from the above said is that down-conversion, channelization, matched filtering, synchronization, and sample rate conversion (SRC) have to be realized digitally. In [1] these functionalities have been combined in the so-called digital front end (DFE). Due to the typically high sample rate as well as the high dynamic range of the signals the functionalities of the DFE are the most power- and time-critical ones of the terminal. Especially filtering operations consume a great deal of the computing resources. In the following it will be seen that nearly all parts of the DFE contain filtering operations which can be merged together. This results in a considerable reduction of the effort. In order to be independent from the requirements of different air-interfaces, as well as to allow for easy parameterization, a novel implementation of polynomial based filters is proposed.

2 Generic software defined receiver
By modifying a conventional receiver an ad hoc solution for a SWR receiver can be derived (see fig. 1). The received signal is amplified and down converted to a lower IF (or direct converted to baseband) by the RF stage respectively. In case of IF (sub)sampling the ADC has to be followed by digital down conversion (DDC). Due to the wideband reception channelization and matched filtering has to be done totally digitally. The need of sample rate conversion (SRC) comes up while performing digitization at a fixed rate. It should be noted that the filter requirements can get very high so that it is inevitable to find new solutions for implementing digital filters [2].

3 Idea of combined filter
The idea of combining the matched filter, and the interpolator (fractional sample delay – FSD) for symbol timing recovery into one FIR filter has been introduced in [3]. As a simple implementation a polyphase approach was proposed (see fig. 2(a)). It was claimed that \( L = 16 \) would be a sufficient number of timing phases / polyphase branches. However, it should be noted that implementing \( L \) polyphase branches is a simple but not very efficient solution since only one branch out of \( L \) is used for calculating an output sample. To avoid this, in [4] matched filtering was combined with polynomial based interpolation using the well known Farrow-structure [5] (see fig. 2(b)).

In this paper we go a step further and add the task of SRC to the polynomial based matched filter / interpolator to enable its application in SWR (see fig. 3). This is possible if the matched filter can be used as a reconstruction fil-
For SRC. Due to the fact that transmit/receive filters are chosen as low pass filters to save bandwidth this constraint is compatible with the combined filter in most cases. Two problems result from this combination. First, when choosing a feed-forward timing-estimator for synchronization it has to cope with the problem that the timing instants of the input samples are non integer multiples of the symbol rate. Second, when implementing the polynomial based filter on the original Farrow structure the length of the concatenated polynomials and thus, also the achievable performance, depends on the input sample rate. In this paper we address the second problem by implementing the filter in a novel way called the transposed Farrow structure [6].

4 Transposed Farrow-Structure

Because sampling in the analog-to-digital converter is done at a fixed rate \(1/T_s\) and independent from the symbol rate \(1/T\), SRC has to performed. The signals at the output and the input of the sample rate converter are related as follows, where \(\tau\) reflects the delay due to non-synchronized sampling, and where \(|\tau - \hat{\tau}|\) should be minimized at the output.

\[
y(mT - (\tau - \hat{\tau})) = \sum_{k=-\infty}^{\infty} x(kT_s - \tau) h(mT - kT_s + \hat{\tau})
\]  

(1)

Usually there are no restrictions on \(T_s/T\) and thus, the complete continuous-time impulse response \(h(t)\) must be known. Even if \(T_s/T\) is rational or can be approximated by \(L/M\) with \(L,M \in \mathbb{N}\), \(L\) and \(M\) can get very large. Therefore, it can be concluded that storing all necessary samples of \(h(t)\) is not sensile. A more practical way is to store just only few samples of \(h(t)\) and calculate all others on demand using polynomials as interpolating functions.

\[
h(t) = \sum_{i=-\frac{K}{2}}^{\frac{K}{2}-1} h_i(t) = \sum_{i=-\frac{K}{2}}^{\frac{K}{2}-1} c_i^T f_j(t)
\]  

(2)

\[
c_i = \begin{bmatrix} c_{0,i} \\ \vdots \\ c_{N,i} \end{bmatrix}, \quad f_j(t) = \begin{bmatrix} f_{0,j}(t) \\ \vdots \\ f_{K,j}(t) \end{bmatrix}
\]  

(3)

\[
f_{j,i}(t) = \begin{cases} \left( \frac{2(i+1)\Delta}{\Delta} \right) - 1 \right)^j & \text{for } i\Delta \leq t < (i+1)\Delta \\ 0 & \text{else} \end{cases}
\]  

(4)

with \(K\) being the number of slices to be concatenated, and \(N\) the highest degree of polynomials. Instead of using polynomials \(f_{j,i}(t)\) in \(t/\Delta\) as in [6] we use them in \(2t/\Delta - 1\) as in [7] to take advantage of symmetry \(h(t) = h(-t)\) halving the number of coefficient multipliers.

\[
c_{j,i} = c_{j,i-1} \quad \text{for } j \text{ even}
\]  

(5)

\[
c_{j,i} = -c_{j,i-1} \quad \text{for } j \text{ odd}
\]  

(6)
Substituting (2)–(4) to (1) leads to a general form which can be simplified for \( \Delta = T_s \) or \( \Delta = T \) (or integer fractions of them). While the first results in the well known Farrow structure [5] the second yields a novel filter structure called the transposed Farrow structure [6] (see fig. 4).

\[
y(mT - (\tau - \hat{\tau})) = \sum_{k=0}^{m} x(kT_s - \tau) c_{j,i,m,k} (2\hat{\mu}(kT_s) - 1)^i
\]

\[
u = \left\lceil \frac{m + \frac{K}{T}}{T_s} \right\rceil
\]

\[
\nu = \begin{cases} \left\lfloor \frac{m - \frac{K}{T}}{T_s} \right\rfloor + 1 & \text{if } \left\lfloor \frac{m - \frac{K}{T}}{T_s} \right\rfloor \in \mathbb{Z} \\ \left\lfloor \frac{m - \frac{K}{T}}{T_s} \right\rfloor & \text{else} \end{cases}
\]

\[
i(m,k) = m - \left\lfloor \frac{kT_s}{T} \right\rfloor \in \left[ -K, K - \frac{1}{2} \right] \]

\[
\hat{\mu}(kT_s) = kT_s/T - \hat{\tau} - \left\lceil kT_s/T - \hat{\tau} \right\rceil \in [0, 1)
\]

The derivation of this structure is comprehensively discussed in a companion paper [6].

While the intersample position \( \hat{\tau} \) is constant for \( T_s/T \in \mathbb{N} \), it is time-varying for non integer ratios \( T_s/T \). So the sequence \( \hat{\mu}(kT_s) \) must be generated. It can be provided by an overflowing accumulator. For \( T_s < T \) (i.e., effective down sampling) it is

\[
\hat{\mu}(kT_s + 1) = \hat{\mu}(kT_s) + \frac{T_s}{T} - \left\lceil \hat{\mu}(kT_s) + \frac{T_s}{T} \right\rceil
\]

When implementing (12) on a DSP or ASIC only a certain precision is available. Hence, \( T_s/T \) must be approximated by a rational factor \( L/M \) always. Therefore, the following investigations deal with rational factor SRC allowing further simplifications. The quantity \( \hat{\mu}(kT_s) = M\hat{\mu}(kT_s) \) is introduced by scaling \( \hat{\mu}(kT_s) \).

\[
\hat{\mu}(kT_s + 1) = (\hat{\mu}(kT_s) + L) \pmod{M}
\]

The same scaling is applied to \( c_{j,i} \) and \( y(mT) \). It simplifies the implementation on with fixed point arithmetic \( (M \) must be even).

\[
\hat{c}_{j,i} = \left( \frac{M}{2} \right)^{N-j} c_{j,i}
\]

\[
\hat{y}(mT) = \left( \frac{M}{2} \right)^{N} y(mT)
\]

The signal \( ov(mT) \) in fig. 4 indicates an overflow of the accumulator

\[
\hat{\mu}(kT_s) + L \geq M
\]

and thus, controls the down sampler as well as the integrate and dump (I&D) circuitry. If \( ov(mT) \) is set, a new output sample is fed into the output tap delay line followed by a reset of the I&D block.

Comparing the original Farrow structure with the transposed one no advantages or disadvantages attract attention at first sight. Still, choosing \( \Delta = T \) instead of \( \Delta = T_s \) means longer polynomial pieces resulting in a smaller number of slices \( K \) to be concatenated for the same transfer characteristics. Since these slices (polynomial pieces) are longer, their order \( N \) must be increased in many cases in order to keep the approximation error low. Despite this drawback the transposed Farrow-structure usually wins over the original Farrow-structure in terms of effort. Another advantage is that the achievable performance is independent from \( T_s \). However, the main advantage is the fact that the transposed Farrow-structure is a decimator. It implements an anti-aliasing filter while the original Farrow-structure is an interpolator which can only implement anti-imaging filters with poor anti-aliasing behavior [6]. Still, the capability of rejecting potential aliasing components is the most important constraint to be fulfilled by any SRC system [1].
5 Filter optimization

Due to the limited precision in real implementations only rational factor SRC by a factor \( L/M \) is performed. This means that only samples of \( h(t) \) at \( t = nT_s/L \) are used. Because \( L \) and \( M \) can get very large and are not always known beforehand it is sensible to implement and optimize the continuous-time impulse response \( h(t) \) using the impulse invariant (II) method instead of trying to optimize samples of \( h(t) \) directly as a digital filter. Only if \( L \) is very small the II method will be disadvantageous.

Optimizing the filter \( h(t) \) requires an error criterion to be minimized. The tasks of \( h(t) \) are matched filtering, anti-aliasing filtering, and also anti-imaging filtering. In the case of a root Nyquist-filter zero inter-symbol-interference can be included. These requirements have to be fulfilled by the desired filter with the impulse-response \( h_u(t) \) and the frequency response and \( H_a(f) \). The square error to be minimized is composed from an error in the time domain and an error in the frequency domain:

\[
E_{\text{tot}}^2 = \alpha E_{\text{time}}^2 + (1 - \alpha) E_{\text{freq}}^2 \rightarrow \min
\]  

with

\[
E_{\text{time}}^2 = \int \left( W(t) |h(t) - h_u(t)|^2 \right) dt
\]

\[
\sum_{i = -\frac{N}{2}}^{\frac{N}{2}} \int \left( W(t) |c_i f_i(t) - h_u(t)|^2 \right) dt
\]

and with \( W(t) \) as weighting function and \( A \) as optimization interval. (Likewise in the frequency domain where \( f \) is replaced by \( f \) and \( f_j(t) \) by its Fourier transform \( F_j(f) \).) Including constraints like symmetry of \( h(t) \) (see (5)), and a continuous shape of \( h(t) \) and its derivatives, yields a set of linear equations with \( c_u \) as independent coefficients

\[
c_i = K_i c_u + l_i
\]

\[
E_{\text{time}}^2 = \sum_{i = -\frac{N}{2}}^{\frac{N}{2}} \int \left( W(t) |c_u T K_i f_i(t) + \alpha_i f_i(t) - h_u(t)|^2 \right) dt
\]

\[
= c_u T \left[ \sum_{i = -\frac{N}{2}}^{\frac{N}{2}} K_i A_i K_i \right] c_u + 2 \left[ \sum_{i = -\frac{N}{2}}^{\frac{N}{2}} \left( \alpha_i A_i - a_i A_i \right) K_i \right] c_u + \\
+ \sum_{i = -\frac{N}{2}}^{\frac{N}{2}} \left( \alpha_i A_i l_i - 2a_i T l_i + b_i \right)
\]

where

\[
A_i = \int_{i \Delta}^{(i+1) \Delta} W_i(t) f_i(t) f_i^T(t) dt
\]

\[
a_i^T = \int_{i \Delta}^{(i+1) \Delta} W_i(t) h_u(t) f_i^T(t) dt
\]

\[
b_i = \int_{i \Delta}^{(i+1) \Delta} W_i(t) h_a^2(t) dt
\]

Then the square error is minimized:

\[
E_A^2 = c_u^T P c_u + 2p^T c_u + q \rightarrow \min
\]

\[
\nabla_{c_u} E_A^2 = 2c_u^T P + 2p = 0
\]

\[
c_u = -P^{-1} p
\]

\[
c_i = -K_i p^{-1} p + l_i
\]

The elements of the vectors \( c_i \) are the sought polynomial coefficients (3) that are directly implemented in the transposed Farrow-structure. With (14) the \( \beta_{ij} \) can be calculated by using the elements of the vectors \( c_i \). A final test should prove that all requirements are fulfilled.

6 Design example

The receive filter \( h_u(t) \) is assumed to be a truncated root Nyquist-filter with a roll off factor \( r = 0.22 \). The ratio of the input and output sample rate is set to \( T = 2.3 T_s \). The length of the impulse response of the combined filter is \( 12 T_s (K = 12) \). The highest degree of the polynomials is \( N = 2 \) for the original Farrow structure resulting in an effort of 20 multipliers. For the transposed Farrow structure the parameters fulfilling the requirements are \( K = 8 \) and \( N = 3 \) resulting in an effort of 19 multipliers. A solution with separate filters would require the transposed Farrow structure with \( K = 8 \) and \( N = 3 \) for SRC as well, while the matched filter with e.g. 11 taps would require 6 additional multipliers. A following linear interpolator for the FSD would require 1 additional multiplier. Fig. 5 and 6 show the resulting responses \( h(t) \) and \( H(f) \) for both, the original and the transposed Farrow-structure. Although both structures have the same complexity the transposed form reaches better performance.

<table>
<thead>
<tr>
<th></th>
<th>passband ripple</th>
<th>aliasing attenu.</th>
</tr>
</thead>
<tbody>
<tr>
<td>original Farrow</td>
<td>±0.4 dB</td>
<td>25 dB</td>
</tr>
<tr>
<td>transposed Farrow</td>
<td>±0.15 dB</td>
<td>28 dB</td>
</tr>
</tbody>
</table>

7 Conclusions

Both, the original as well as the transposed Farrow structure are suitable choices for implementing parameterizable SRC. Due to its anti-aliasing properties the transposed form allows a further reduction of complexity compared to
the original Farrow-structure. Adding the tasks of matched filtering and fractional sample delay is possible, and compared to the separate implementation of these tasks the overall effort can be decreased considerably.

**Literature**


**Biographies**

Matthias Henker (henker@ifn.et.tu-dresden.de) received his MSc/Dipl.-Ing. degree from the Dresden University of Technology, Germany, in November 1998. In his diploma thesis he analyzed algorithms for sample rate conversion in software programmable mobile communications receivers.

In December 1998 he joined the Mannesmann Mobilfunk Chair for Mobile Communications Systems at the Dresden University of Technology, Germany. His main research interests include Software Radio, especially Dual- and Multimode Terminals.
Gerhard Fettweis (fettweis@ifn.et.tu-dresden.de) received his MSc/Dipl.-Ing. and PhD. degree in electrical engineering from the Aachen University of Technology (RWTH), Germany, in 1986 and 1990, respectively.

From 1990 to 1991 he was a Visiting Scientist at the IBM Almaden Research Center in San Jose, CA, working on signal processing for disk drives. From 1991 to 1994 he was Scientist with TCSI, Berkeley, CA, responsible for signal processor developments for mobile phones. Since September 1994 he holds the Mannesmann Mobilfunk Chair for Mobile Communications Systems at the Dresden University of Technology, Germany.

He is an elected member of the SSC Society’s Administrative Committee, and of IEEE ComSoc Board of governors, since 1999 and 1998, respectively. He has been associate editor for IEEE Trans. on CAS II, and now is associate editor for IEEE J-SAC wireless series.