Energy-Efficient Link Adaptation with Transmitter CSI

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Abstract—Energy-efficient link adaptation is studied based on minimizing the total energy consumption per transmitted bit in a mobile terminal. It is shown that the optimal power allocation is water-filling; the optimal energy consumption per bit is a function of the power amplifier efficiency, the circuit power rate dependence, and a cutoff channel to noise ratio (CNR). For a given transceiver architecture, low energy consumption per bit corresponds to a high cutoff CNR and vice versa. The optimization of the energy consumed per bit is carried out for flat fading and frequency-selective fading channels. In the former, the optimization problem is solved analytically, resulting in optimal rate, power and energy consumption per bit being expressed in terms of the cutoff CNR. In the latter, the optimal power allocation is water-filling in frequency, depending on the subcarrier noise levels. Finally, highly efficient algorithms with superlinear convergence are proposed, which are based on the Dinkelbach method for solving nonlinear fractional programs.

I. INTRODUCTION

Energy efficiency in mobile communication devices is becoming increasingly important since the battery capacity is unable to keep up with increasing power dissipation of signal processing circuits [1]. Improving the energy efficiency during the active transmission involves minimizing the energy consumption per bit [2] or equivalently maximizing the “throughput per Joule” metric [3].

Link adaptation based on channel state information (CSI) is normally used to maximize throughput for a given total transmission power. However, it can also be directed towards maximizing energy efficiency. Earlier work on energy-efficient link adaptation has modeled the power dissipated in a mobile terminal during transmission as the sum of power dissipated in the processing circuit and the transmission power scaled by a power amplifier inefficiency parameter. Energy-efficient link adaptation has been studied previously for channels exhibiting flat fading [3] and frequency-selective fading [4], [5].

This work provides more insight into previous results for both the flat-fading channel, where an analytical solution is found, and the frequency-selective case. Moreover, highly efficient algorithms for solving the optimization problems are proposed, based on the Dinkelbach method for solving general nonlinear fractional programs. The algorithms exhibit superlinear convergence.

The rest of this paper is organized as follows. In Section II, the optimization problems are formulated for flat fading and frequency-selective fading channels, respectively. In Section III, these problems are treated mathematically based on results from the literature on fractional programming. The results are discussed in Section IV and Section V concludes the paper.

II. PROBLEM FORMULATION

A. Flat fading channel

Consider a flat fading AWGN channel with bandwidth B. The channel to noise ratio (CNR) of the channel is defined as \( \gamma = \frac{|h|^2}{N_0} \), where \( h \) is the complex channel coefficient experienced by the channel and \( N_0 \) is the noise power spectral density. Under the assumption that perfect CSI is available at both transmitter and receiver, the maximum rate (in bits/s) that can be reliably transmitted over the channel is

\[
r = B \log_2 \left( 1 + \frac{\gamma P}{B} \right),
\]

where \( P \) is the transmitted power. Solving for \( P \), we get

\[
P(r) = \left( 2^{r/B} - 1 \right) \frac{B}{\gamma},
\]

which is convex and continuously differentiable in \( r \). For energy-efficient communication, we wish to minimize the energy consumption per bit (denoted by \( E_a \)), which corresponds to dissipated power divided by the throughput, thus the problem to be solved can be stated as

\[
\text{minimize}_{r \in \mathbb{R}_+} E_a = \frac{P_C(r) + \varepsilon P(r)}{r},
\]

where \( P_C \) is the circuit power dissipation, \( \varepsilon \) is a parameter that expresses power amplifier inefficiency.

The circuit power \( P_C \) includes all transmitter circuitry including baseband processing and radio frequency (RF) transceiver frontend. It is a common assumption that the power dissipation in a chip can be adequately modeled as the sum of a static term and a dynamic term,

\[
P = V_{dd} \cdot I_{leak} + a \cdot f \cdot C \cdot V_{dd}^2,
\]

where \( a \) is related to the effective fraction of gates switching, \( f \) is the clock frequency, and \( C \) is the circuit capacitance. If the frequency is dynamically scaled with the rate, it is reasonable to model the power dissipation as a linear function of the rate with a constant offset, as supported by earlier publications [6].
constant [2]. Based on this, we model the circuit power as

\[ P_C = \alpha + \beta r. \]

The parameter \( \varepsilon \) is given by the peak-to-average power ratio (PAPR) divided by the drain efficiency of the power amplifier [2]. In this paper, \( \varepsilon \) is assumed to be a constant.

**B. Frequency-selective channel**

Consider a parallel AWGN channel consisting of a set of \( K \) non-interfering subcarriers, where the noise is independent across subcarriers. With a frequency-selective, block fading channel, the CNR of subcarrier \( i \) is defined as \( \gamma_i = \frac{|h_i|^2}{N_0} \), where \( h_i \) is the complex channel coefficient experienced by the subcarrier \( i \) and \( N_0 \) is the noise power spectral density. Assuming perfect CSI at both transmitter and receiver, the problem to be solved is stated as

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Based on this, we model the circuit power as \( P \) constant [2]. Based on this, we model the circuit power as \( P \) constant [2]. Based on this, we model the circuit power as \( P \) constant [2]. In this paper, \( \varepsilon \) is assumed to be a constant.

\[ C = \begin{bmatrix} C_{11} & \cdots & C_{1K} \\ \vdots & \ddots & \vdots \\ C_{K1} & \cdots & C_{KK} \end{bmatrix}, \]

\[ P = \begin{bmatrix} P_1 \\ \vdots \\ P_K \end{bmatrix}, \]

\[ P_r = \begin{bmatrix} P_{r1} \\ \vdots \\ P_{rK} \end{bmatrix}, \]

\[ r_i = W \log_2 \left( 1 + \frac{\gamma_i P_i}{W} \right), i = 1, \ldots, K, \]

where \( W \) denotes the subcarrier spacing and \( P_i \) is the subcarrier transmission power. Solving for \( P_i \), we get

\[ P_i = \frac{(2^{r_i/W} - 1) W}{\gamma_i}, i = 1, \ldots, K, \]

which is convex and continuously differentiable in \( r_i \). The problem to be solved is stated as

**III. MATHEMATICAL ANALYSIS**

For both channel models discussed above, the optimization problems is a convex-concave fractional program, where the numerator is convex in \( r \) and the denominator is affine (convex as well as concave). It is well known that the objective function is quasiconvex [7]. By negating the numerator, an equivalent concave-convex fractional program with a quasi-concave objective function is obtained. Since concave-convex fractional programs share some important properties with concave programs [8], it is possible to solve concave-convex fractional programs with many of the standard methods for concave programs.

Dinkelbach [9] treated the following problem

\[ \max_{x \in S} q(x) = \frac{f(x)}{g(x)}, \]

where \( S \) is a compact, connected set and \( g(x) > 0 \) is assumed. Since the rates are always bounded in a real system, it is clear that the sets in the problems discussed in this paper are compact. The concave-convex fractional program above can also be associated with the following parametric concave program [8]

\[ \max_{x \in S} f(x) - qg(x) \]

where \( q \in \mathbb{R} \) is treated as a parameter. Problem (5) might be mathematically more tractable than the concave-convex fractional problem since it is concave and can be shown to yield the same set of solutions. The optimal value of the objective function in the parametric problem, henceforth denoted by \( F(q) \), is a convex and continuous function that is strictly decreasing.

Let \( x^* \) be an optimal point in (5) and \( q^* = \frac{f(x^*)}{g(x^*)} \). Then the following statements are equivalent [9]

\[ F(q) > 0 \iff q < q^*, \]

\[ F(q) = 0 \iff q = q^*, \]

\[ F(q) < 0 \iff q > q^*. \]

Thus, solving problem (5) is equivalent to finding the root of the nonlinear equation \( F(q) = 0 \).

The algorithm described in Fig. 1 (known as the Dinkelbach method) is in fact the application of Newton’s method to a nonlinear fractional program [8]. Therefore, the sequence converges to the optimal point with a superlinear convergence rate. A detailed convergence analysis can be found in [10]. The initial point can be any \( q_0 = \frac{f(\tilde{x})}{g(\tilde{x})} \) with a feasible \( \tilde{x} \) that satisfies \( F(q_0) \geq 0 \).

**A. Flat fading channel**

For the flat fading channel, the parametric concave optimization problem can be stated as

\[ \max_{r \in \mathbb{R}^K} -(P_C(r) + \varepsilon P(r)) - q \cdot r, \]

where \( q \in \mathbb{R} \) is treated as a parameter, corresponding to \( -E_a \) in problem (2). Here, the optimal value of the objective function,

\[ F(q) = F(q, r^*(q)) = -(P_C(r^*) + \varepsilon P(r^*)) - qr^*, \]

is a convex, continuous and strictly decreasing function of \( q \). The problem above needs to be solved in each step of Dinkelbach’s algorithm.

Since it is obvious that a strictly feasible point exists for problem (6), strong duality holds according to Slater’s condition [11] and the Karush-Kuhn-Tucker (KKT) conditions are
necessary and sufficient for optimality. The KKT conditions are in this case

\[ r^* \geq 0, \]
\[ \beta + \frac{\varepsilon P}{dr}|_{r=r^*} + q = 0. \]

From the last row, we have

\[ -q - \beta = \frac{\varepsilon P}{dr}|_{r=r^*}. \]

Differentiating (1) and inserting into this expression yields

\[ -q - \beta = \varepsilon \ln 2 \cdot 2^{-r^*/B} \cdot \frac{1}{\gamma}. \] (7)

Solving for \( r^* \), we get

\[ r^* = B \left( \log_2 \frac{-q - \beta}{\varepsilon \ln 2} - \log_2 \frac{1}{\gamma} \right). \]

Note that \( q < -\beta \) is needed for the first term to be real. Moreover, if \( \frac{1}{r^*} > \frac{q - \beta}{\varepsilon \ln 2} \), the resulting \( r^* \) would be negative, so in this case \( r^* = 0 \). Thus, we have the optimal rate

\[ r^*(q) = B \cdot \max \left( \log_2 \frac{-q - \beta}{\varepsilon \ln 2} - \log_2 \frac{1}{\gamma}, 0 \right), \] (8)

with the corresponding optimal power given by (1),

\[ P^*(q) = B \cdot \max \left( \frac{-q - \beta}{\varepsilon \ln 2} - \frac{1}{\gamma}, 0 \right). \] (9)

By introducing a cutoff CNR

\[ \gamma_0 = \frac{\varepsilon \ln 2}{-q - \beta}, \]

which is a function of the parameters \( q, \beta \) and \( \varepsilon \), the optimal rate and power can alternatively be expressed as

\[ r^*(\gamma_0) = B \log_2 \left( \frac{\gamma}{\gamma_0} \right) \]

and

\[ P^*(\gamma_0) = B \left( \frac{1}{\gamma_0} - \frac{1}{\gamma} \right), \]

respectively, provided \( \gamma \geq \gamma_0 \). The channel is not used whenever the instantaneous CNR falls below the cutoff value \( \gamma < \gamma_0 \). The optimal power is therefore given by water-filling [12], as illustrated in Figure 2.

Once the optimal rate \( r^* \) and power \( P^* \) have been calculated for a given value of the parameter \( q \) (or, equivalently, for a given cutoff CNR \( \gamma_0 \)), the next step is to determine the optimal energy consumption per bit iteratively using the modified version of the Dinkelbach method described in Fig. 3.

For this simple channel model, in fact it is possible to derive the optimal value \( q^* = -E_{\gamma}^* \) in closed form. Remember that we wish to find the solution to the nonlinear equation \( F(q) = 0 \), i.e.

\[ P_C(r^*) + \varepsilon P(r^*) + q^* \cdot r^* = 0, \]

or

\[ -q^* - \beta = \frac{\alpha + \varepsilon P(r^*)}{r^*}. \] (10)

Fig. 2. Water-filling power allocation, indicated by the double-headed arrow between the curve \( 1/\gamma \) and a cutoff level \( 1/\gamma_0 \). The black line shows the location of the cutoff CNR \( \gamma_0 \) on the horizontal CNR axis.

**Require:** \( q_0 \) satisfying \( F(q_0) \geq 0 \), tolerance \( \Delta \)

\( n \leftarrow 0 \)

**repeat**

Calculate \( r^*_n \) from (8) and \( P^*_n \) from (9)

\( q_{n+1} \leftarrow \left( \beta + \frac{\alpha + \varepsilon P^*_n}{r^*_n} \right) \)

\( n \leftarrow n + 1 \)

**until** \( |F(q_n)| \leq \Delta \)

Fig. 3. The Dinkelbach method for energy-efficient link adaptation on a flat fading channel as modeled by optimization problem (2).

Introducing

\[ y = 2^{-r*/B} = \frac{\gamma}{\gamma_0}, \]

and combining (10) with (7), we have

\[ \varepsilon \ln 2 \cdot \frac{y}{\gamma} = \frac{\alpha}{\varepsilon} \cdot \frac{\gamma}{B} - 1, \]

where the expression for \( P \) from (1) has been utilized. This can be transformed to

\[ (\ln y - 1) \cdot y = \frac{\alpha}{\varepsilon} \cdot \frac{\gamma}{B} - 1, \]

or

\[ (\ln y - 1)e^{(\ln y - 1)} = x \cdot e^{-1}, \]

where

\[ x = \frac{\alpha}{\varepsilon} \cdot \frac{\gamma}{B} - 1 \]

has been introduced for brevity. The solution to this equation is

\[ \ln y - 1 = W(x \cdot e^{-1}), \]

where \( W \) is the Lambert W function [13]. Note that the condition \( r^* \geq 0 \) yields \( y \geq 1 \), which implies \( W(x \cdot e^{-1}) \geq -1 \), i.e. the principal branch of \( W \) should be selected.
The values of \( r^* \), \( P^* \), and \( q^* \) can now be expressed as follows,

\[
\begin{align*}
r^* &= B \log_2 y \\
P^* &= B(y - 1) \\
q^* &= -\left( \beta + \frac{\epsilon \ln 2}{\gamma} \right) = -\left( \beta + \frac{\epsilon \ln 2}{\gamma_0} \right),
\end{align*}
\]

where

\[ y = \exp(1 + W(x \cdot e^{-1})) = \frac{x}{W(x \cdot e^{-1})}, \quad (11) \]

From (11), we see that \( y > 1 \) (i.e., the condition for \( r^* > 0 \)) requires \( x > W(x \cdot e^{-1}) \), which is satisfied for \( x > 0 \) or equivalently

\[ \gamma > \frac{\epsilon B}{\alpha}. \]

The results on optimal rate and energy consumption per bit as functions of bandwidth are shown in Figure 4. It is obvious that the energy consumption per bit decreases and the rate increases when more bandwidth is available.

Although the analytical solution provides insight and intuition, it may still be attractive to use the Dinkelbach algorithm for numerical evaluation of a given channel realization since numerical evaluation of the Lambert W function also relies on a root-finding algorithm.

**B. Frequency-selective channel**

For the frequency-selective channel, the parametric concave problem is stated as

\[
\max_{r \in \mathbb{R}^K_+} - (P_C(r) + \epsilon 1^T P(r)) - q 1^T r,
\]

where \( q \in \mathbb{R} \), corresponding to \( -E_a \), is treated as a parameter. Again, the optimal value of the objective function, denoted by \( F(q) \), is a convex, continuous and strictly decreasing function of \( q \) and this problem needs to be solved in each step of Dinkelbach’s algorithm.

Referring to Slater’s condition, the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for optimality. The KKT conditions are

\[
\begin{align*}
\gamma^* &\geq 0, & i = 1, \ldots, K \\
\beta + \frac{\epsilon \partial P_i}{\partial r_i^*} + q &= 0, & i = 1, \ldots, K.
\end{align*}
\]

The bottom line yields

\[ -q - \beta = \frac{\epsilon \partial P}{\partial r_i^*}. \]

Inserting the expression for \( P_i \) from (3), we obtain

\[ -q - \beta = \frac{\epsilon dP_i}{\partial r_i^*} = \epsilon \ln 2 \cdot 2 r_i^*/W \cdot \frac{1}{\gamma_i}. \]

Solving for \( r_i^* \), we get

\[ r_i^* = W\left( \log_2 \frac{-q - \beta}{\epsilon \ln 2} - \log_2 \frac{1}{\gamma_i} \right), i = 1, \ldots, K. \]

Note that \( q < -\beta \) is needed for the first term to be real. Moreover, if \( \frac{1}{\gamma_i} > \frac{\epsilon}{\ln 2} \), the resulting \( r_i^* \) would be negative, so in this case \( r_i^* = 0 \). Thus, we have the optimal rate allocation

\[ r_i^* = W \cdot \max\left( \log_2 \frac{-q - \beta}{\epsilon \ln 2} - \log_2 \frac{1}{\gamma_i}, 0 \right), i = 1, \ldots, K, \quad (13) \]

and the subcarrier power allocation given by (3),

\[ P_i^* = W \cdot \max\left( \frac{-q - \beta}{\epsilon \ln 2} - \frac{1}{\gamma_i}, 0 \right), i = 1, \ldots, K. \quad (14) \]

The optimal subcarrier power allocation corresponds to water-filling in the frequency domain, as illustrated in Figure 5.
and powers can be written as since the function $F$ are also shown. Note that the algorithm converges to the root from below since $F$ is convex. The explicit solution to optimization problem (12) derived from (13) and (14) is

$$
q^* = W \log_2 \left( \frac{\gamma_i}{\gamma_0} \right)
$$

and

$$
P^* = W \left( \frac{1}{\gamma_0} - \frac{1}{\gamma_i} \right),
$$

respectively, where $\gamma_i$ is the CNR of subcarrier $i$.

The explicit solution to optimization problem (12) derived above is used as the basis for a modified version of the Dinkelbach method for energy-efficient link adaptation on the frequency-selective channel, as shown in Fig. 6. The performance of the algorithm is illustrated in Figure 7. The algorithm, which is based on Newton’s method, converges to the root from below since $F(q)$ is convex. This example illustrates the superlinear convergence that result in a fairly accurate estimate of the solution after only two iterations.

IV. DISCUSSION

We have seen that energy-efficient link adaptation results in water-filling, where the parameter $q$, corresponding to the (negative) energy consumption per bit, is a function of the parameters $\varepsilon$ and $\beta$ of the transceiver system as well as the cutoff CNR $\gamma_0$. Since the parameters $\varepsilon$ and $\beta$ can be assumed to be fixed depending on the actual transceiver architecture, the optimal $q^*$ is a function of the cutoff CNR. A high energy consumption per bit translates to a low cutoff CNR and vice versa. The optimal rate $r^*$ and optimal power $P^*$ can also be expressed in terms of the cutoff CNR. Below a certain CNR $\gamma = \frac{\alpha}{\beta}$, the most energy-efficient strategy is not transmitting at all. Above this level, the optimal rate and power increase with the CNR.

For the frequency-selective, block fading channel, the optimal power allocation is water-filling in frequency. The power allocated to a certain subcarrier depends on the normalized noise-to-carrier ratio of each subcarrier as well as on the common cutoff CNR.

V. CONCLUSION

We have shown that energy-efficient link adaptation is based on water-filling, where the energy consumption per bit is determined by the cutoff level. If the channel is frequency-selective, water-filling in frequency should be carried out according to the subcarrier noise levels. The optimal cutoff level can be found by solving the energy efficiency optimization problem using a modified version of the Dinkelbach method, which is in fact the Newton method for nonlinear optimization and therefore exhibits superlinear convergence. The energy consumption per bit decreases with increasing CNR and with increasing bandwidth.

ACKNOWLEDGMENT

The authors would like to thank their colleagues at Technische Universität Dresden for various suggestions. This work was financed by BMBF (German Ministry of Education and Research) through the Cool Silicon Cluster of Excellence.

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