Energy-Efficient Link Adaptation with Shadow Fading

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Abstract—Energy-efficient link adaptation is studied for a channel exhibiting log-normal shadow fading in addition to path loss. Both circuit power and power dissipated in the power amplifier are considered. The effective throughput per Joule of energy is maximized over transmit power and a margin for shadowing. It is demonstrated that the optimization problem can be transformed into a pair of concave maximization problems over transmit power and shadowing margin, respectively. Effective algorithms with superlinear convergence are proposed to solve these concave problems. The effects of varying the circuit power, the power amplifier inefficiency parameter or the distance between transmitter and receiver are discussed. It is shown that neglecting the shadowing component by setting the shadowing margin to zero may result in large losses in energy efficiency.

I. INTRODUCTION

Energy efficiency in mobile communication devices is becoming increasingly important since the battery capacity is unable to keep up with increasing power dissipation of signal processing circuits [1]. Link adaptation based on channel state information (CSI) is normally used to maximize throughput for a given total transmission power. However, it can also be directed towards maximizing energy efficiency.

Earlier work on energy-efficient link adaptation [2], [3] has modeled the power dissipated in a mobile terminal during transmission as the sum of power dissipated in the processing circuit and transmission power scaled by a power amplifier inefficiency parameter. Improving the energy efficiency during the active transmission involves minimizing the energy consumption per bit [2] or equivalently maximizing the throughput-per-Joule metric [3].

In earlier work the signal path loss from transmitter to receiver was assumed to be perfectly known. In this work, a system with combined path loss and shadow fading is considered. Shadowing refers to random variations in the received power around the mean distance-dependent path loss due to blockage from objects as well as changes in reflecting surfaces and scattering objects. As empirically confirmed in both indoor [4] and outdoor [5] radio propagation environments, the shadowing component can be modeled as a zero-mean random variable with log-normal distribution. The transmitter is assumed to have accurate knowledge of the path loss but only statistical knowledge of the shadowing component. Thus, for any finite transmission rate, there is a nonzero probability that the value of the random shadow fading is such that the transmitted data cannot be reliably decoded at the receiver. It is therefore natural to ask what margin for random shadowing the transmitter should allow for when setting the rate.

The rest of this paper is organized as follows. The system model is described in Section II. The problem is formulated and mathematically analyzed in Section III. Simulation results are discussed in Section IV and Section V concludes the paper.

II. SYSTEM MODEL

The total power dissipation at the transmitter side is modeled [6] as $P_C + \varepsilon P_t$, where $P_C$ is the circuit power, $P_t$ is the transmission power (both in W/Hz), and $\varepsilon \geq 1$ is a power amplifier inefficiency parameter. The circuit power $P_C$ includes all transmitter circuitry including baseband processing and radio frequency (RF) transceiver frontend and is modeled as a constant here. The parameter $\varepsilon$ is given by the output back-off (OBO) divided by the drain efficiency of the power amplifier [2] and is assumed to be a constant as well.

Path loss is defined as the power reduction (in dB) from the transmitter to the receiver. The average path loss at distance $d$ has the well-known form

$$\overline{PL}(d) = \overline{PL}_0 + 10n \log_{10} (d/d_0),$$

where $\overline{PL}_0$ is the path loss at the reference distance $d_0$ and $n$ is the path loss exponent.

When expressed logarithmically (in dB), the power attenuation from shadowing is normally distributed with mean zero and standard deviation $\sigma$. Thus, when combined with shadow fading, the path loss model becomes

$$PL(d) = \overline{PL}(d) + S,$$

where $S$ is a zero-mean Gaussian random variable with standard deviation $\sigma$. Alternatively, $S$ can be written as $S = \sigma y$, where $y$ is a Gaussian random variable with zero mean and unit variance.

The channel to noise ratio (CNR) is defined as $\gamma = |h|^2/N_0$, where $h$ is the complex channel coefficient and $N_0$ is the noise power spectral density. The average channel to noise ratio (CNR) resulting from the distance dependent mean path loss then becomes

$$\overline{\gamma} = 10^{-\overline{PL}(d)/10}/N_0.$$

The combined effects of path loss and shadowing lead to a random received CNR

$$\gamma = \overline{\gamma} \cdot 10^{-S/10} = \overline{\gamma} \cdot e^{-ky},$$
where $k = \sigma \log 10/10$.

It is assumed that the path loss can be accurately estimated, meaning that $\bar{\gamma}$ is known at the transmitter side. The statistical characterization of the shadowing (i.e. $\sigma$) is also assumed to be known, whereas the random shadowing at the receiver is unknown.

III. Problem Formulation

Since the instantaneous shadowing is not known at the transmitter side, the transmitter encodes for a rate

$$R = \log_2 (1 + \gamma_0 P_t),$$

where

$$\gamma_0 = \bar{\gamma} \cdot 10^{-\sigma y_0/10} = \bar{\gamma} \cdot e^{-k y_0}$$

is a cutoff CNR, which in turn is specified by the design parameter $y_0$, which corresponds to a margin for shadowing. The transmission is successful if the instantaneous CNR is higher than the cutoff CNR, or equivalently if the instantaneous shadowing $S$ is lower than $\sigma y_0$. Thus, we have

$$1 - p_{\text{out}} = \mathbb{P} \{ S < \sigma y_0 \} = \mathbb{P} \{ y < y_0 \} = \Phi(y_0),$$

where $p_{\text{out}}$ is the outage probability and $\Phi$ is the cumulative distribution function of the normal distribution,

$$\Phi(x) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{x}{\sqrt{2}} \right) \right] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt.$$

Figures 1 and 2 show how the rate $R$ and the outage probability $p_{\text{out}}$, respectively, vary as functions of the parameter $y_0$. The effective rate is given by

$$R(1 - p_{\text{out}}) = \log_2 (1 + \gamma_0 P_t \cdot e^{-\kappa y_0}) \cdot \Phi(y_0).$$

For energy-efficient transmission we wish to maximize the effective rate divided by the total power dissipation. Therefore, the problem to be solved is

$$\max_{y_0 \in \mathbb{R}, \; \gamma_0 \in \mathbb{R}} \frac{R(1 - p_{\text{out}})}{P_C + \varepsilon P_t} = \frac{\log_2 (1 + \gamma_0 P_t \cdot e^{-\kappa y_0}) \cdot \Phi(y_0)}{P_C + \varepsilon P_t}. \tag{2}$$

It is instructive to investigate the asymptotic behavior of the objective function for extreme values of $P_t$ and $y_0$. For $P_t$ small, the numerator is close to zero whereas the denominator is finite (close to $P_C$). Therefore, the maximum is not expected to occur for $\gamma_0 P_t << 1$ or, equivalently, $P_t << N_0 \cdot 10^{10B \varepsilon (d + \sigma y_0)/10}$. Conversely, the denominator increases linearly with $P_t$ whereas the numerator increases only logarithmically. Therefore, the objective function tends to zero for large $P_t$.

For $10^{-\sigma y_0/10} << 1/\bar{\gamma}$, the expression $\log(1 + \gamma_0)$ is close to zero while $\Phi(y_0)$ is close to one. The product is therefore close to zero. Conversely, when $10^{-\sigma y_0/10} >> 1/\bar{\gamma}$, the expression $\log(1 + \gamma_0)$ depends linearly on $y_0$ whereas $\Phi(y_0)$ tends exponentially to zero. Therefore the product will tend to zero when $y_0$ is large and negative.

A typical contour plot of the energy efficiency as a function of $P_t$ and $y_0$ is shown in Fig. 3. As will be discussed in detail below, the objective function is a quasiconcave function of $P_t$ whereas it is a log-concave function of $y_0$. Since the optimization problem is not concave in the joint variable $(P_t, y_0)$, we focus on restating this problem as a concave maximization problem in each of the variables separately.

A. Optimization over shadowing margin

We first turn our attention to optimization over $y_0$, with $P_t$ given. The numerator in (2) contains a product of two functions of $y_0$. The cumulative distribution function of the normal distribution is known to be log-concave [7], meaning that

$$f_2(y_0) = \log \Phi(y_0)$$

is concave. It is therefore natural to take the logarithm of the energy efficiency expression,

$$\max_{y_0 \in \mathbb{R}} f(y_0) = \log \log_2 (1 + \gamma_0) + \log \Phi(y_0) - \log (P_C/B + \varepsilon P_t),$$
where the last term is independent of $y_0$. If we can show that the function above is concave in $y_0$, we know that the stationary point is the global maximum. Therefore, we need to show that

$$f_1 = \log \log_2(1 + \gamma_0) = \log \log(1 + \gamma_0) + \log \log_2 e,$$

where $\gamma_0 = \bar{\gamma} \cdot e^{-ky_0}$, is concave in $y_0$.

The first and second derivatives with respect to $y_0$ can be found using the chain rule as follows,

$$\frac{df_1}{dy_0} = \frac{df_1}{d\gamma_0} \cdot \frac{d\gamma_0}{dy_0} = \frac{1}{(1 + \gamma_0) \log(1 + \gamma_0)} \cdot \frac{1}{1 + \gamma_0} \cdot (-k\gamma_0) =$$

$$\frac{d^2 f_1}{dy_0^2} = \frac{d}{d\gamma_0} \left( \frac{df_1}{dy_0} \right) = \frac{(1 + \gamma_0) \log(1 + \gamma_0) - \gamma_0(1 + \gamma_0) + 1}{(1 + \gamma_0)^2} \cdot k^2 \gamma_0 =$$

$$= \frac{\log(1 + \gamma_0) \log(1 + \gamma_0) - \gamma_0(1 + \gamma_0)^2}{(1 + \gamma_0)^2} \cdot k^2 \gamma_0 < 0,$$

because $\log(1 + \gamma_0) < \gamma_0$ for any $\gamma_0 > 0$. Since $e^x > 0$ for all $x \in \mathbb{R}$, we conclude that $\frac{d^2 f_1}{dy_0^2} < 0$ for all $y_0 \in \mathbb{R}$, hence $f_1$ is concave in $y_0$, as required.

Setting $f'(y_0) = 0$ yields

$$\frac{df}{dy_0} = \frac{df_1}{dy_0} + \frac{df_2}{dy_0} = \frac{-k\gamma_0}{(1 + \gamma_0) \log(1 + \gamma_0)} + \frac{\phi(y_0)}{\Phi(y_0)} = 0,$$

where $\phi$ is the probability density function of the normal distribution. This equation is not possible to solve analytically because of the non-elementary error function in $\Phi(y_0)$.

Instead, based on the second order derivative,

$$\frac{d^2 f}{dy_0^2} = \frac{d^2 f_1}{dy_0^2} + \frac{d^2 f_2}{dy_0^2},$$

where the second term is given by

$$\frac{d^2 f_2}{dy_0^2} = \frac{\Phi(y_0) \phi'(y_0) - \phi(y_0) \Phi'(y_0)}{\Phi(y_0)^2} =$$

$$= \frac{\Phi(y_0) \cdot (\gamma_0 \phi(y_0) - \phi(y_0) \Phi(y_0))}{\Phi(y_0)^2} =$$

$$= -\frac{\phi(y_0)}{\Phi(y_0)} \left( y_0 + \frac{\phi(y_0)}{\Phi(y_0)} \right),$$

the solution can be calculated numerically with Newton’s method [7] as described in Fig. 4.

The result of this algorithm when used for different values of $P_t$ is shown by the blue curve in Fig. 5. As expected, the curve always crosses the level curves perpendicularly at the leftmost and rightmost points, respectively.

### B. Optimization over transmit power

When maximizing over $P_t$ instead (with $y_0$ given), the problem is stated as

$$\max_{P_t \in \mathbb{R}} \frac{\log_2(1 + \gamma_0 P_t) \Phi(y_0)}{N_0 + \epsilon P_t},$$

where

$$\gamma_0 = \bar{\gamma} \cdot e^{-k y_0}.$$
In this case the objective function is a ratio of a concave function in the numerator and an affine (hence convex) function in the denominator. This kind of optimization problem is called a concave-convex fractional program and the objective function is known to be quasiconcave [8]. One way of solving a concave-convex fractional program is to transform it to an equivalent parametric concave program, in this case as follows,

\[
\max_{P_t \in \mathbb{R}_+} \log_2(1 + \gamma_0 P_t) \Phi(y_0) - q(P_C + \varepsilon P_t),
\]

where \( q \in \mathbb{R} \) is treated as a parameter. The optimal value of the objective function in the parametric problem (4), henceforth denoted by \( F(q) \), is a convex, continuous and strictly decreasing function of \( q \) [8].

Let \( P_t^* \) be the solution to (4) and

\[
q^* = \frac{\log_2(1 + \gamma_0 P_t^*) \Phi(y_0)}{P_C + \varepsilon P_t^*}.
\]

Then the following statements are equivalent [9]

\[
\begin{align*}
F(q) &> 0 \iff q < q^*, \\
F(q) &= 0 \iff q = q^*, \\
F(q) &< 0 \iff q > q^*.
\end{align*}
\]

Thus, solving problem (4) is equivalent to finding the root of the nonlinear equation \( F(q) = 0 \).

The algorithm described in Fig. 6 (known as the Dinkelbach method [9]) is in fact the application of Newton’s method to a nonlinear fractional program [8]. Therefore, the sequence converges to the optimal point with a superlinear convergence rate. A detailed convergence analysis can be found in [10].

The initial point in the algorithm can be any

\[
q_0 = \frac{\log_2(1 + \gamma_0 \tilde{P}_t) \Phi(y_0)}{P_C + \varepsilon \tilde{P}_t}
\]

with a feasible \( \tilde{P}_t \) that satisfies \( F(q_0) \geq 0 \).

The solution to problem (4) is found by setting the first derivative of the objective function with respect to \( P_t \) equal to zero:

\[
\log_2 e \cdot \frac{\gamma_0}{1 + \gamma_0 P_t^*} \cdot \Phi(y_0) - q \varepsilon = 0.
\]

Since \( \varepsilon \geq 1 \), we can rearrange the expression to obtain

\[
P_t^* = \frac{\log_2 e \cdot \Phi(y_0)}{q \varepsilon} - \frac{1}{\gamma_0}. \tag{5}
\]

The result of the algorithm in Fig. 6 when used for different values of \( y_0 \) is shown by the red curve in Fig. 5. Note that the curve always crosses the level curves perpendicularly at the top and bottom points, respectively.

The two algorithms can also be used interchangeably, with the resulting \( P_t^* \) from the algorithm in Fig. 4 used as parameter value in the algorithm in Fig. 6 and vice versa. Since each restricted optimization is global, this alternating optimization is able to “hop” great distances through the variable space in each iteration and may therefore be suitable for bypassing local minima [11]. As illustrated in Fig. 5, this leads to a fast convergence for various starting points (in this case \( y_0 = 0 \)).

For the simplest case that no shadowing margin is allowed for, we have \( y_0 = 0 \), \( \gamma_0 = \gamma \) and \( \Phi(y_0) = 0 \). In this case, the equation \( F(q) = 0 \) can be solved analytically, resulting in the expression

\[
q = \frac{\gamma \log_2 e \cdot W\left(\frac{\gamma P_C}{\varepsilon - 1}\right)}{2(\gamma P_C - \varepsilon)}, \tag{6}
\]

where \( W \) is the Lambert W function [12]. The corresponding transmit power is given by

\[
P_t = \frac{\log_2 e - 1}{2 \gamma} \cdot \frac{1}{\gamma} \tag{7}
\]

Taking the constraint \( P_t \geq 0 \) into account, it turns out that the principal branch \( W_0 \) should be selected in (6).

### IV. Simulation results

The simulations are based on the experimental indoor path loss curves in [4]. The plot shown in Fig. 3 and Fig. 5 uses the parameters listed in Table I and the environment LOS Commercial in Table II. The optimized parameter values for the various environments are given in the last two columns in Table II.

By varying one of the parameters while keeping the other one constant, the isolated effect of each parameter can be studied. For example, Fig. 7 depicts how the situation changes when the circuit power \( P_C \) is increased to 1 W. The blue curve is the same since the optimization over \( y_0 \) is not affected by the numerator in (2). The optimum energy efficiency is lower and occurs at a point further to the right along the blue curve, with higher values for both \( P_t \) and \( y_0 \). Similarly, a higher

<table>
<thead>
<tr>
<th>Environment</th>
<th>( P_{\text{LO}} )</th>
<th>( n )</th>
<th>( \sigma )</th>
<th>( P_t^* )</th>
<th>( y_0^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS Commercial</td>
<td>43.7 dB</td>
<td>2.97</td>
<td>2.3</td>
<td>176 mW</td>
<td>0.285</td>
</tr>
<tr>
<td>NLS Commercial</td>
<td>47.3 dB</td>
<td>2.95</td>
<td>4.1</td>
<td>554 mW</td>
<td>-0.138</td>
</tr>
<tr>
<td>LOS Residential</td>
<td>45.9 dB</td>
<td>2.01</td>
<td>3.2</td>
<td>197 mW</td>
<td>0.285</td>
</tr>
<tr>
<td>NLS Residential</td>
<td>50.3 dB</td>
<td>3.12</td>
<td>3.8</td>
<td>975 mW</td>
<td>-0.073</td>
</tr>
</tbody>
</table>

### TABLE I

**SIMULATION PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>( P_{\text{LO}} )</td>
<td>0.1 W</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>1/0.35</td>
</tr>
<tr>
<td>( d_0 )</td>
<td>1 m</td>
</tr>
<tr>
<td>( d )</td>
<td>10 m</td>
</tr>
<tr>
<td>( N_0 )</td>
<td>( 10^{-7} ) W/Hz</td>
</tr>
</tbody>
</table>

### TABLE II

**RESULTS FOR UWB PATH LOSS MODEL FROM [4].**

<table>
<thead>
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<th>Environment</th>
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value of the power amplifier inefficiency parameter $\varepsilon$ shifts the optimum along the blue curve to the left, while the resulting energy efficiency decreases.

The result when the distance $d$ is increased from 10 m to 20 m is shown in Fig. 8. As expected, the optimum energy efficiency occurs for a higher value of $P_t$ and is lower than in Fig. 5.

Although the energy efficiency loss using the simple expressions in (6) and (7) is small when $y_0^*$ happens to be close to zero as in the environment NLS Residential (in this case 2.5%), the loss is dramatic in other cases (80% for the environment LOS Commercial).

**V. Conclusion**

We have seen that the original energy efficiency maximization problem can be transformed to a pair of concave maximization problem for optimization over shadowing margin and over transmit power, respectively. Alternating optimization can be used to find the optimum energy efficiency. The optimization over shadowing margin is independent of the parameters used to model the transmitter power dissipation. A higher circuit power leads to a shift towards the right along the curve for optimal shadowing margin as a function of transmit power and to a reduced energy efficiency. Similarly, with a less efficient power amplifier the optimum shifts towards the left along the same curve. The energy efficiency is reduced also in this case. Neglecting the influence of shadowing by setting the shadowing margin to zero may result in large losses in energy efficiency.

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**References**