Generalized Mutual Information of MIMO SC-FDMA with Mismatched Receivers

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Abstract—We investigate simplified, yet mismatched receivers for SC-FDMA MIMO transmission, currently employed in the 3GPP LTE-Advanced uplink. The receivers are based on frequent domain equalisation and time domain detection. Rates achievable by those mismatched receivers with arbitrary input alphabets are derived in terms of generalized mutual information. The results allow to compare the impact of certain simplifying receiver assumptions under various side conditions, such as spatial correlation or channel length. In addition, the results could be used to adjust the transmission rate according to the receiver detection metric.

I. INTRODUCTION

Discrete Fourier transform (DFT)-spread single carrier frequency division multiplex (SC-FDMA) is an attractive technique to achieve low peak-to-average power ratios, required to operate transmit amplifiers efficiently, which is particularly important for battery powered mobile terminals. SC-FDMA has, therefore, been selected for application in the 3GPP-LTE-Advanced uplink [1] combined with per-antenna coded MIMO techniques. MIMO SC-FDMA transmission turns out to be equivalent to a MIMO transmission over an inter-symbol interference (ISI) channel which, generally, requires more complex receivers, as compared to, e.g., MIMO-OFDM due to the inherent channel memory. Quite recently, the corresponding detection problem has received increasing attention and to adapt transmission rates for different receivers.

The remainder of this paper is structured as follows: the SC-FDMA transmission model is introduced in Section V. Encoding, mismatched decoding, and GMI are treated in Section III. Different receivers and their achievable rates are presented in Section IV and numerically compared in Section V. Section VI concludes the paper.

Notation: $\mathcal{N}_C(\mu, \phi)$ denotes the complex normal distribution (mean $\mu$, covariance $\phi$). Normal ($\mu$) and boldface ($\mathbf{a}$) letters denote scalars and vectors or matrices respectively. $Pr(\cdot)$, $p(\cdot)$ and $\mathbb{E}_a[\cdot]$ denote a probability, a probability density function (pdf) and the expectation with regard to random vector $a$. $\mathbf{1}_T$, $(\cdot)^T$ and $(\cdot)^H$ denote the identity matrix of size $T \times T$, the matrix transpose, and the hermitian operator, respectively. $\mathbf{F}_M \in \mathbb{C}^{M \times M}$ denotes a Fourier matrix with elements $[\mathbf{F}_M]_{m1,m2} = \exp(-j2\pi m1m2/M)$.

II. SC-FDMA TRANSMISSION WITH FREQUENCY DOMAIN EQUALIZATION

We present a brief summary of MIMO SC-FDMA transmission and then specialize important parts for further use. The detailed derivation of the SC-FDMA discrete-time base-band matrix transmission model is tedious but straightforward and is, hence, omitted.

A. Transmission Model

We investigate a MIMO system with $N_T$ transmit and $N_R$ receive antennas, shown in Fig. 1. $M$ time domain data symbols comprised in a vector $\mathbf{x}_{j,t} \in \mathbb{C}^{MN_T \times 1}$, $\mathbf{x}_j' = [x_j'[1], \ldots, x_j'[M]]^T$ are to be transmitted from antenna $t$ in SC-FDMA symbol $j$. The time domain signal vector $\mathbf{x}_j \in \mathbb{C}^{MN_T \times 1}$, $\mathbf{x}_j' = [x_j'[1], \ldots, x_j'[M]]^T$ captures all symbols to be transmitted from the $N_T$ transmit antennas. $\mathbf{x}_j$ is composed of zero-mean MIMO channel input vectors $x_j[m] = [x_{j,1}[m], \ldots, x_{j,N_T}[m]]^T$ with covariance matrix $\mathbf{\Sigma}_j^T = \mathbf{I}_{N_T}$, $\forall m_1 = m_2$ and $0$ elsewhere.

In a SC-FDMA system, the time domain symbol vector $\mathbf{x}_{j,t}$
is transformed into frequency domain by means of a DFT, followed by a mapping onto \( N \geq M \) subcarriers of an OFDM symbol. The time domain signal is computed by an inverse DFT (IDFT) and subsequent pre-pending of a cyclic prefix of length \( N_{CP} \) followed by a transmission over a time dispersive channel. The channel is characterized by its impulse response \( \tilde{h}_{r,t} \in \mathbb{C}^{N \times 1} \) with the first \( L \leq N_{CP} + 1 \) elements being non-zero and the remaining elements being equal to zero. We assume that the channel impulse response remains static during \( J \) SC-FDMA symbols. The cyclic prefix turns the linear convolution with the channel impulse response into a cyclic convolution.

The received signal is perturbed by zero-mean additive white Gaussian noise with variance \( \sigma^2_w \). The synchronized receiver deletes the cyclic prefix, followed by a DFT and a selection of the \( M \) subcarriers occupied by the SC-FDMA symbol. The received frequency domain vector of size \( N_R \times 1 \) at subcarrier \( m \) is equalized (FDE) by left-multiplication with a matrix \( \sigma_w h \) in order to mitigate frequency selectivity and hence ISI as well as spatial interference. The \( t \)-th output of the \( M \) equalizers is fed into an I-DFT, yielding the relation

\[
\tilde{y}_{j,t} = \tilde{h}_j \tilde{x}_{j,t} + \tilde{\nu}_{j,t}
\]

between the time domain channel input \( \tilde{x}_{j} \) and the time domain effective channel output \( \tilde{y}_{j} \). \( \tilde{\nu}_{j,t} \) in (1) denotes zero-mean time domain Gaussian receiver noise including the effect of frequency domain equalization.

It can be easily shown that the effective channel impulse response matrix \( \tilde{h}_j \in \mathbb{C}^{MN_R \times MN_T} \) (including frequency domain equalization) has a block-cyclic structure. In particular, \( \tilde{h}_j \) is composed as follows

\[
\tilde{h}_j = \begin{bmatrix}
\tilde{h}_j[1] & \tilde{h}_j[M] & \cdots & \tilde{h}_j[3] & \tilde{h}_j[2] \\
& \ddots & \ddots & \ddots & \ddots \\
\tilde{h}_j[M] & \tilde{h}_j[M-1] & \cdots & \tilde{h}_j[2] & \tilde{h}_j[1]
\end{bmatrix},
\]

with \( \tilde{h}_j[m] \in \mathbb{C}^{N_R \times N_T} \) and its elements derived from the I-DFT of the frequency domain channel coefficients after equalization as follows

\[
\begin{bmatrix}
\tilde{h}_{t_1,t_2}[1], \ldots, \tilde{h}_{t_1,t_2}[M]
\end{bmatrix}^T = \frac{1}{M} F^H_M \begin{bmatrix}
\tilde{H}_{t_1,t_2}[1], \ldots, \tilde{H}_{t_1,t_2}[M]
\end{bmatrix}^T.
\]

\( \tilde{H}_{t_1,t_2}[m] = [\tilde{H}[m]]_{t_1,t_2} \) is an element of the frequency domain MIMO channel matrix \( \tilde{H}[m] \in \mathbb{C}^{N_T \times N_T} \) after equalization \( \tilde{H}[m] = W[m]H[m] \). The MIMO channel coefficients contained in \( H[m] \) are obtained from the DFT of the channel impulse response as follows

\[
[H_{r,t}[1], \ldots, H_{r,t}[N]]^T = F_N \tilde{h}_{r,t}.
\]

Throughout the numerical evaluation we employ minimum mean square error (MMSE) equalization. \( W[m] \), in that case, reads

\[
W[m] = H^H[m] \left( H[m]H^H[m] + \frac{\sigma^2_w}{\sigma^2_N} \frac{MN_T}{N} I_{N_T} \right)^{-1}.
\]

In order to derive the covariance matrix of the zero-mean additive Gaussian noise \( \tilde{\nu}_j \) in (1), denote \( \tilde{w}_j[m] \) as \( \tilde{w}_j[m] = W[m]W[m]^H \in \mathbb{C}^{N_R \times N_T} \). From the I-DFT of its elements

\[
[w_{t_1,t_2}[1], \ldots, w_{t_1,t_2}[M]]^T = F^H_M \tilde{w}_{t_1,t_2}[1], \ldots, \tilde{w}_{t_1,t_2}[M]^T
\]

we obtain the elements of the block-cyclic noise covariance matrix

\[
\phi_{\tilde{\nu}_j} = \mathbb{E} [\tilde{\nu}_j[1] \tilde{\nu}_j^H] = \frac{\sigma^2_w}{N} \begin{bmatrix}
\end{bmatrix}.
\]

with \( W[m] \in \mathbb{C}^{N_R \times N_T} \) and \( \tilde{w}_{t_1,t_2}[m] \).

B. Model Properties

From (1) it follows that cyclic prefixed MIMO SC-FDMA transmission with frequency domain equalization equals a single-carrier MIMO transmission over a cyclic ISI channel, i.e., a channel with memory, characterized by the Gaussian transition pdf \( p(\tilde{y}_j | \tilde{x}_j, \tilde{h}_j) \sim N_c(\text{h}_j, \sigma^2_\nu). \) It can be easily shown that MMSE-FDE per subcarrier equals time domain MMSE equalization of the complete received block \( \tilde{y}_j = \tilde{w}_{\text{time}} + \tilde{w}_{\text{time}} \) with \( \tilde{w}_{\text{time}} = \arg\min_{\tilde{w}_{\text{time}}} \left\{ ||\tilde{w}_{\text{time}} - \tilde{y}_j||^2 \right\} \) where \( \tilde{y}_j \) is equal to \( \tilde{y}_j \) in (1) with \( W[m] \) replaced by an identity matrix. From (1), we can rewrite the channel input-output relation regarding the \( m \)-th MIMO channel input vector.
as shown below

\[ \hat{y}_j[m] = \mathbf{h}[1|x_j[m] + \sum_{\tau=0}^{M-1} \mathbf{h}([(M - \tau + m - 1) \mod M] + 1|x_j[\tau + 1] + \bar{v}[m]} \]

(2)

The corresponding transition pdf is derived by computing the marginal density

\[ p\left(\hat{y}_j[m] | x_j[m], \mathbf{h}\right) = \sum_{\mathbf{z}_j[m]} p\left(\hat{y}_j[m] | x_j[m], \mathbf{z}_j[m], \mathbf{h}\right) \]

(3)

with regard to all, except the \( m \)-th, MIMO channel inputs \( \mathbf{z}_j[m] \). Computing this marginal density involves large sums, depending on the input signal distribution. Low-complexity receivers may be devised by modeling the residual ISI, contained in \( \mathbf{z}_j[m] \), to be Gaussian distributed, which reduces (2) to the standard model for MIMO transmission over a narrow-band block-static MIMO channel with spatially correlated noise

\[ \hat{y}_j[m] \approx \mathbf{h}[1|x_j[m] + \mathbf{z}_j[m], \mathbf{z}_j[m] \sim \mathcal{N}\left(0, \phi_{xx}\right). \]

(4)

The noise covariance matrix in (4) is

\[ \phi_{xx} = \sigma^2/N_r \sum_{m=1}^{M} \mathbf{h}[m] \mathbf{h}[m]^H + \sigma^2/N_r \mathbf{w}_m^H \mathbf{w}_m. \]

(5)

Note that the approximation (4) is accurate if the input signals \( \mathbf{x}_j[m] \) were Gaussian distributed.

### III. MISMATCHED DECODING AND GENERALIZED MUTUAL INFORMATION

We are interested in deriving achievable rates when independently encoding the \( N_r \) channel inputs over \( J \) SC-FDMA symbols and using the mismatched receivers. Therefore, denote the complete received block \( \mathbf{y} = \left[ \mathbf{y}_1^T, \ldots, \mathbf{y}_J^T \right]^T \) and the complete encoded symbol vector at antenna \( t: \mathbf{y}_t = \left[ \mathbf{x}_1^T, \ldots, \mathbf{x}_{N_r}^T \right]^T \). The transition pdf for a complete encoded signal block decomposes according to

\[ p\left(\mathbf{y}_t | \mathbf{x}_1, \ldots, \mathbf{x}_{N_r}\right) = \prod_{j=1}^{J} p\left(\mathbf{y}_j | \mathbf{x}_j, \mathbf{x}_1, \ldots, \mathbf{x}_{N_r}\right) \]

(5)

which characterizes a memoryless vector channel. In the following random coding discussion we specialize to the case of two transmit antennas. The methodology, however, can be extended straightforwardly.

Assume, data source one and two at antenna one and two, each randomly select one out of \( A \) and \( B \) messages, indexed \( a \) and \( b \), with equal probability. These messages are mapped by encoding functions to transmit sequences \( \mathbf{x}_a \) and \( \mathbf{x}_b \) drawn from the codebooks \( \{ \mathbf{x}_a^1, \ldots, \mathbf{x}_a^A \} \) and \( \{ \mathbf{x}_b^1, \ldots, \mathbf{x}_b^B \} \). The corresponding code rates in nats per symbol are \( R_1 = \log(A)/J/M \) and \( R_2 = \log(B)/J/M \), respectively.

Subsequently, we closely follow Gallager’s random coding arguments [10] applied to multiple-access channels [7], [8]. That is, we upper-bound the probability of decoding error, averaged over the ensemble of codebooks. The codebooks are generated by independently drawing \( A \) sequences according to the probability \( \text{Pr}(\mathbf{x}_a^m) = \prod_{j=1}^J \text{Pr}(x_j^m) \) from the symbol alphabet \( \mathcal{X}_1 \) and \( B \) sequences according to the probability \( \text{Pr}(\mathbf{x}_b^m) = \prod_{j=1}^J \text{Pr}(x_j^m) \) from the symbol alphabet \( \mathcal{X}_2 \).

A maximum likelihood decoder decides for the message pair \( \hat{a}, \hat{b} \) if \( p\left(\mathbf{y}_t | \mathbf{x}_a, \mathbf{x}_b\right) > p\left(\mathbf{y}_t | \mathbf{x}_{\hat{a}}, \mathbf{x}_{\hat{b}}\right) \) for any message pair \( a', b' \neq a, b \). The mismatched decoder we are interested in uses a decoding metric \( q(\mathbf{y}_t | \mathbf{x}_{\hat{a}}, \mathbf{x}_{\hat{b}}) \) instead of \( p\left(\mathbf{y}_t | \mathbf{x}_{\hat{a}}, \mathbf{x}_{\hat{b}}\right) \) and decides for the message pair \( \hat{a}, \hat{b} \) if \( q(\mathbf{y}_t | \mathbf{x}_{\hat{a}}, \mathbf{x}_{\hat{b}}) > q(\mathbf{y}_t | \mathbf{x}_{a'}, \mathbf{x}_{b'}) \) for any message pair \( a', b' \neq a, b \).

Three types of error can occur: 1) \( e_1: \hat{a} \neq a, \hat{b} = b \), 2) \( e_2: \hat{a} = a, \hat{b} \neq b \) and 3) \( e_3: \hat{a} \neq a, \hat{b} \neq b \). From [7], [8], the probability of the first error event \( e_1 \), conditioned on message \( a \) has been encoded into \( \mathbf{x}_a, \mathbf{x}_b \) was transmitted from the second antenna and receiving \( \mathbf{y} \) can be upper-bounded by (maximum likelihood decoding, \( \alpha \) denotes \( \mathbf{x}_a, \mathbf{x}_b \))

\[ \text{Pr}(e_1) \leq \left( (A - 1) \sum_{\mathbf{x}_{a'}} \text{Pr}(\mathbf{x}_{a'}) \frac{p(\mathbf{y}_t | \mathbf{x}_a, \mathbf{x}_b)}{p(\mathbf{y}_t | \mathbf{x}_{a'}, \mathbf{x}_b)} \right) ^{s_1} \]

(6)

with \( 0 \leq p_1 \leq 1 \) and \( s_1 > 0 \). The term (*) is derived by upper-bounding [10]

\[ \sum_{\mathbf{x}_{a'}} \text{Pr}(\mathbf{x}_{a'}) \leq \sum_{\mathbf{x}_{a'}} \text{Pr}(\mathbf{x}_{a'}) \frac{q(\mathbf{y}_t | \mathbf{x}_{a'}, \mathbf{x}_b)}{q(\mathbf{y}_t | \mathbf{x}_a, \mathbf{x}_b)} \]

where the summation range on the LHS captures all sequences \( \mathbf{x}_b \) which cause an erroneous decision of the maximum likelihood decoder. Similarly, [9] modified this bounding technique to be applicable with a mismatched receiver as follows

\[ \sum_{\mathbf{x}_{a'}} \text{Pr}(\mathbf{x}_{a'}) \leq \sum_{\mathbf{x}_{a'}} \text{Pr}(\mathbf{x}_{a'}) \frac{q(\mathbf{y}_t | \mathbf{x}_{a'}, \mathbf{x}_b)}{q(\mathbf{y}_t | \mathbf{x}_a, \mathbf{x}_b)} \]

which then replaces (*) in (6).

Analogous to the matched receiver case [8], i.e., by averaging (6) over all conditioning variables, exploiting the memoryless channel property\(^3\) (5) and noting that the message indices \( a \) and \( b \) are dummy variables, the average probability of the first error event\(^2\) is upper bounded by

\[ \mathcal{T}_{e,1} \leq (A - 1)^{s_1} \mathbb{E} \left[ \sum_{\mathbf{x}_{a'}} \text{Pr}(\mathbf{x}_{a'}) q(\mathbf{y}_t | \mathbf{x}_a, \mathbf{x}_b)^{s_1 p_1} \right] \]

(7)

\(^2\) It is assumed that the mismatched metric decomposes into a product over the \( J \) vector channel uses \( \prod_j q(\mathbf{y}_j | \mathbf{x}_a^j, \mathbf{x}_b^j) \) similar to (5).

\(^3\) The index \( j \) is omitted.
Likewise, the average probability of the error events $e_2$ and $e_3$ are upper-bounded by

$$
\mathbb{P}_{e,2} \leq (B - 1)^{\rho^2} \mathbb{E} \left[ \sum_{x_1, x_2} \Pr(x_1) q(\tilde{y}, x_1, x_2)^{s_2 \rho^2} \right]^{J}
$$

and

$$
\mathbb{P}_{e,3} \leq (A - 1)^{\rho^3} (B - 1)^{\rho^3} \mathbb{E} \left[ \sum_{x_1, x_2} \Pr(x_1) \Pr(x_2) q(\tilde{y}, x_1, x_2)^{s_3 \rho^3} \right]^{J}
$$

with $s_{2,3} > 0$ and $0 \leq \rho_{2,3} \leq 1$. Defining $E_0, (p_i, s_i) = -\log(f(p_i, s_i))$, all three error probabilities can be put in the common exponential form

$$
\mathbb{P}_{e,i} \leq \exp \left( -J (E_0, (p_i, s_i) - p_i R_i) \right),
$$

where $R_{i=3} = R_1 + R_2$ denotes the sum rate. The error probability can be made arbitrary small by increasing $J$ as long as

$$
R_i \leq \frac{1}{M} \frac{\partial E_0, (p_i, s_i)}{\partial p_i} \bigg|_{p_i=0} = \frac{1}{M} I_G(s_i)
$$

holds, where the GMI $I_G(s_i)$ derives by computing the partial derivative in (7) as

$$
I_G(s_i) = \mathbb{E} \left[ \log \left( \frac{q(\tilde{y}, x_1, x_2)}{u(s_i)} \right) \right],
$$

with

$$
\begin{align*}
    u(s_1) & = \sum_{x_1} \Pr(x_1) q(\tilde{y}, x_1, x_2)^{s_1} \\
    u(s_2) & = \sum_{x_2} \Pr(x_2) q(\tilde{y}, x_1, x_2)^{s_2} \\
    u(s_3) & = \sum_{x_1, x_2} \Pr(x_1, x_2) q(\tilde{y}, x_1, x_2)^{s_3}.
\end{align*}
$$

An extension to arbitrary many channel inputs follows straightforwardly. A sum rate $M(R_1 + \ldots + R_{NT})$ is achievable if it is chosen smaller than the generalized mutual information

$$
I_G(s_4) = \mathbb{E} \left[ \log \left( \frac{q(\tilde{y}, x_1, \ldots, x_{NT})}{u(s_4)} \right) \right]
$$

with

$$
\begin{align*}
    u(s_4) & = \sum_{x_1, \ldots, x_{NT}} \Pr(x_1, \ldots, x_{NT}) q(\tilde{y}, x_1, \ldots, x_{NT})^{s_4}.
\end{align*}
$$

The GMI constitutes a lower bound on the achievable rate [9], which can be improved by maximization over $s_i$. For matched receivers $s_i = 1$ holds [10]. (8) establishes a means to compare different mismatched receivers, i.e., different choices of $q(\cdot)$ in terms of the corresponding achievable rates in the next section.

IV. SIMPLIFIED RECEIVERS AND ACHIEVABLE RATES

We focus on the sum rate according to (9) and apply the GMI to two different mismatched receiver types.

A. The Matched Receiver

The matched receiver, which will serve as an upper bound, jointly decides for the sequences which maximize the transition pdf (5). The maximum achievable sum rate is determined by the mutual information $I(\cdot; \cdot)$

$$
\frac{1}{M} I(\tilde{y}; x_1, \ldots, x_{NT})
$$

$$
= \frac{1}{M} \mathbb{E} \left[ \log \left( \frac{p(\tilde{y}, x_1, \ldots, x_{NT})}{p(\tilde{y})} \right) \right]
$$

$$
= \frac{1}{M} \mathbb{E} \left[ \log \left( \det \left( \text{I}_{MN_T} + \frac{\sigma^2}{N_T} \hat{h} h^H \phi_{\tilde{y}} \right) \right) \right].
$$

The expectation is computed with regard to $\tilde{y}, x_1, \ldots, x_{NT}$. The second line holds for Gaussian distributed input signals.

B. Memoryless Channel Assumption

A first step towards low-complexity receivers is to disregard the channel memory and to treat residual ISI as noise. That is, the receiver jointly detects the $N_T$ channel input sequences, assuming that the transition pdf of a vector channel use (one SC-FDMA symbol) decomposes into a product over the $M$ channel uses as shown below

$$
q(\tilde{y}, x_1, \ldots, x_{NT}) = \prod_{m=1}^{M} p(\tilde{y} | x_1[m], \ldots, x_{NT}[m]),
$$

where $p(\tilde{y} | x_1[m], \ldots, x_{NT}[m])$ is obtained as defined in (3). This assumption would be accurate if the channel was not time dispersive. Inserting (10) into (9) we obtain

$$
\frac{1}{M} I_G(s_4)
$$

$$
= \mathbb{E} \left[ \log \left( \frac{p(\tilde{y} | x_1, \ldots, x_{NT})}{\prod_{m=1}^{N_T} \Pr(x_m) p(\tilde{y} | x_1, \ldots, x_{NT})} \right) \right]
$$

$$
= I(\tilde{y}; x_1, \ldots, x_{NT}).
$$

(11) equals the mutual information of a memoryless MIMO channel with matched receiver. The GMI is hence maximized by choosing $s_4 = 1$. In case of Gaussian distributed channel inputs\(^4\) (12) yields

$$
I(\tilde{y}; x_1, \ldots, x_{NT}) = \log \left( \det \left( \text{I}_{NT} + \frac{\sigma^2}{N_T} \hat{h} \hat{h}^H \phi_{\tilde{y}}^{-1} \right) \right).
$$

\(^4\) With Gaussian channel inputs the summation over the channel inputs in the denominator of (11) is appropriately replaced by integration.
(12) can be expanded into a form, well suited for numerical evaluation. A detailed expression is given in [11, eq. (9)].

C. Memoryless Channel Assumption and Parallel Detection

From a receiver implementation perspective, it may be desired to detect the \( N_T \) channel input sequences in parallel rather than jointly, allowing to exploit parallel receiver structures and minimizing detection delay [11]. To implement parallel detection, the receiver assumes the \( N_T \) channel inputs to be independent, conditioned on the received signal. I.e., the receiver bases its decision on the mismatched metric\(^5\)

\[
q (\mathbf{y}, \mathbf{x}_1, \ldots, \mathbf{x}_{N_T}) = \prod_{m=1}^{N_T} \prod_{t=1}^{T} p (\mathbf{y}[m]|x_t[m]).
\]  

(13)

This assumption would be accurate if the MIMO channel matrix was orthogonal. Inserting (13) into (9) yields

\[
\frac{1}{M} I_G(s_k) = \sum_{t=1}^{N_T} \mathbb{E} \left[ \log \left( \frac{p (\mathbf{y} | x_t)^{s_k}}{\sum_{s_i} \Pr (x_t)p (\mathbf{y} | x_t)^{s_i}} \right) \right] 
\]

(14)

equals the mutual information of a memoryless channel with matched receiver, with Gaussian receiver noise and additional discrete noise, originating from ISI and spatial interference (other channel inputs). Again, the choice \( s_k = 1 \) hence maximizes the GMI. With Gaussian distributed input signals, the GMI (15) reads

\[
\sum_{t=1}^{N_T} I (\mathbf{y}; x_t) = \sum_{t=1}^{N_T} \log \left( \frac{\det \left( I_{N_T} + \mathbb{E} \hat{h}[I]\hat{h}[I]^H \phi_{zz}^{-1} \right)}{\det \left( I_{N_T} + \frac{\pi^2}{N_T} \hat{h}[I]\hat{h}[I]^H \phi_{zz}^{-1} \right)} \right).
\]

(15)

where \( \hat{h}[I] \in \mathbb{C}^{N_T \times N_T - 1} \) is derived from \( \hat{h}[I] \) by discarding its \( t \)-th column. Note that the parallel receiver equals a linear receiver for the particular choice of Gaussian distributed input signals\(^6\). (IV-C) can hence be viewed as an upper bound on the linear receiver performance with discrete alphabets. However, for the parallel receiver, which can accurately account for the the statistical properties of spatial interference (as opposed to the linear receiver), Gaussian input signals are not necessarily the best choice. In the numerical examples it will be shown, that (IV-C) can be significantly exceeded over a wide SNR range by appropriately choosing discrete modulation alphabets. Similar to the presented in IV-B it becomes almost infeasible to evaluate (14) for discrete alphabets. Hence, we resort to the Gaussian ISI approximation according to (4) in that case. (14) then expands as given by eq. (12) in [11].

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\(^5\) When considering transmission on bit level, (13) could be approximately implemented using a sphere search algorithm as presented in [5].

\(^6\) This equivalence is due to the fact that \( \hat{y} \) can be viewed as the output of the time domain MMSE equalizer. For Gaussian channels and with Gaussian input signals, \( \mathbf{y}_t[m] \) represents a sufficient statistic with regard to the channel input \( x_{j,t}[m] \) in that case.
conditional independency assumption becomes increasingly inaccurate with increasing spatial correlation. The loss of the Mem. less receiver increases as well, since spatial correlation also influences the ability of the frequency domain equalizer to suppress the frequency selectivity of the channel, which results in an increased residual ISI level and a strongly correlated noise covariance matrix $\Phi_{\Sigma \Sigma}$.

### C. GMI with Discrete M-QAM Inputs

We pick a single example channel realization from the parameter set $(L = 9, \rho_s = 0.9)$, which causes severe ISI and strongly ill conditioned\(^7\) MIMO channel coefficients in order to illustrate differences in the receiver performance.

GMI results for Gaussian and M-QAM input signals are shown in Fig. 4. The GMI performance is upper bounded by the optimal $Matched$ receiver and lower bounded by the Mem. less par receiver, both with Gaussian input signals. The loss (a) between the $Matched$ and the Mem. less receiver is caused by the channel memory. It could be mitigated, e.g., by applying different design criteria for FDE. The Mem. less receiver GMI with M-QAM alphabets closely follows the GMI with Gaussian alphabets, but remains lower since Gaussian alphabets maximize the mutual information of the memoryless MIMO channel in (4). By contrast, constantly using 64-QAM instead of Gaussian input signals increases the mutual information of the Mem. less par receiver, which takes advantage of the discrete spatial interference structure. However, a large gap, as compared to the Mem. less receiver remains. This gap can be significantly reduced (c), by applying adaptive modulation (max-QAM), i.e., choosing the combination of modulation schemes which maximizes the GMI, depending on the SNR and the channel properties.

### VI. CONCLUSIONS

We modeled a per-antenna coded MIMO SC-FDMA transmission, currently used in the 3GPP-LTE Advanced uplink, as a transmission over a vector-valued memoryless multiple-access channel. Within the model, each vector channel use corresponds to a SC-FDMA symbol. The model inherently covers point-to-point and multi-point-to-point MIMO transmission. The model allows to state achievable rates of reduced-complexity mismatched receivers in terms of generalized mutual information. Our results highlight that receivers which detect the different channel input sequences in parallel and disregard the channel memory can substantially benefit from adapting the modulation alphabets. The presented methodology is suited to derive achievable rates of other receiver types, such as FDE with TD successive interference cancelation as well. It could also be applied for adapting the transmission rates, coupled to the different receivers.

### REFERENCES