Abstract—In this paper we investigate MIMO spatial multiplexing transmission over slowly time-variant, block-static fading channels with linear receivers. The system can adapt the transmission rate per spatial layer based on outdated channel state information. Two approximate low complexity approaches, based on the log-transformed post equalization SINR, are presented to adjust the transmission rate adaptively on a per block basis. Both approaches can be applied in order to maintain a certain target outage probability or to maximize the mutual information with outage. The methodology is suited for arbitrary input alphabets.

I. INTRODUCTION

Modern wireless communications systems, extensively relying on multi-antenna techniques, exploit the channel quality by a fast adaptation of the transmission parameters to the instantaneous channel state. This is enabled by fast feedback of information about e.g. precoding parameters and the supported transmission rate from the receiver to the transmitter and appears feasible as long as the channel changes slowly as compared to the duration of a transmitted block and the feedback delay. A variety of methods to determine the transmission parameters, e.g. mutual information based methods such as [1], have been developed, mostly based on the assumption of instantaneous and hence accurate feedback. However, feedback delays on the order of milliseconds inevitably occur due to the time required for channel measurement at the receiver, feedback encoding, transmission and adaptation at the transmitter. From an adaptation perspective, this delay introduces uncertainty about the channel quality and hence the supported transmission rate during a future transmission if the channel is time variant. Even a small uncertainty may result in transmission errors and the transmission rate needs to be adapted such that a certain target error probability is achieved.

In that regard, it is important to exploit the correlation of the channel state information (CSI) available during adaptation and the CSI during the actual transmission in order to not assign rates overly conservative.

Much progress has been made concerning rate adaptation for SISO or multi-antenna combining systems [2]–[4]. Those results rely on the accurate knowledge of the channel quality statistics, conditioned on outdated CSI. However, no such work appears to be available for multiple-input-multiple-output (MIMO) spatial multiplexing systems with linear receivers since the channel quality statistics, conditioned on outdated CSI appear to be unknown to the best of the authors knowledge. In the context of linear receivers (LR), channel quality can be measured in terms of post equalization signal-to-interference and noise ratio (SINR). In this work, we present two approaches to model the SINR statistics, conditioned on the outdated CSI which is available during adaptation. Both approaches rely on the log-transformed SINR and yield compact, closed-form approximate expressions for the outage probability per spatial layer. Those expressions can be easily exploited in order to adjust the transmission rate such that it exceeds the mutual information of the actual transmission with a target outage probability.

The remainder of the paper is structured as follows. The transmission setup is introduced in section II. Section III derives a rate adaptation method for narrow-band MIMO systems, which is evaluated in section IV. The work is summarized along with further research directions in V.

Notation: \( N(\mu, \sigma^2) \) and \( \mathcal{N}(\mu, \sigma^2) \) denote the normal and complex normal distribution (mean \( \mu \), variance \( \sigma^2 \)). Normal letters \( a \), lower-case \( a \) and upper-case \( A \) bold face letters denote scalars, vectors and matrices. \( \text{Pr}(\cdot), p(\cdot), F(\cdot) \) and \( \mathbb{E}_a[\cdot] \) denote a probability, a probability density function, a cumulative distribution function and the expectation with regard to random vector \( a \). \( I_T \), \( (\cdot)^T \) and \( (\cdot)^H \) denote the identity matrix of dimension \( T \times T \), the matrix transpose and the hermitian operator, respectively.

II. TRANSMISSION SETUP

A. System Model

We investigate a spatial multiplexing transmission over a narrow-band MIMO channel, represented by a matrix \( \mathbf{H} \in \mathbb{C}^{R \times T} \), using \( T \) transmit and \( R \geq T \) receive antennas. The elements of the channel input vector \( \mathbf{x} \in \mathbb{C}^{T \times 1} \) with covariance \( \sigma^2_{\mathbf{x}} \mathbf{I}_T \) are drawn from independently encoded symbol sequences. That is, each channel input carries a separate code word and the rate per channel input can be adjusted individually. The channel output vector \( \mathbf{y} \in \mathbb{C}^{R \times 1} \) writes

\[
\mathbf{y} = \frac{1}{\sqrt{T}} \mathbf{H} \mathbf{x} + \mathbf{v},
\]  

(1)

where \( \mathbf{v} \sim \mathcal{N}(0, \sigma^2_{\mathbf{v}} \mathbf{I}_R) \) denotes additive white Gaussian receiver noise. The normalization \( \frac{1}{\sqrt{T}} \) models a constant transmit power, regardless of the number of antennas. The signal-to-noise-ratio (SNR) is defined as \( \gamma = \frac{\sigma^2_{\mathbf{x}}}{T \sigma^2_{\mathbf{v}}} \). Block static fading is assumed, i.e., the channel matrix \( \mathbf{H} \) is assumed to remain static during the transmission of a complete block, but is allowed to change in between blocks (see II-B).
A linear receiver decouples the channel inputs, superimposed by the MIMO channel, into a vector $\tilde{x} \in \mathbb{C}^{T \times 1}$ by multiplying the received signal vector $y$ with an equalizer matrix. The equalizer could be designed to completely suppress spatial interference (zero forcing, ZF) or to minimize the mean square error between channel input and equalizer output (MMSE). Thereby, the receiver is assumed to possess perfect knowledge of the MIMO channel matrix $\mathbf{H}$. The resulting $T$ sub-channels $x_t \rightarrow \tilde{x}_t$, which are treated as independent during detection of the transmitted data, are characterized by their post equalization SINR

$$
\tilde{\gamma}^{\text{ZF}}_t = \frac{1}{(\mathbf{H}^H \mathbf{H})^{-1}}_{t,t}
$$

(2)

$$
\tilde{\gamma}^{\text{MMSE}}_t = \frac{1}{(\mathbf{H}^H \mathbf{H} + \gamma^{-1} \mathbf{I}_T)^{-1}}_{t,t} - 1.
$$

(3)

The mutual information [5] resulting with a linear receiver

$$
I^{\text{LR}}(\mathbf{x}; \mathbf{x} | \mathbf{H}) = \sum_{t=1}^{T} I(\tilde{x}_t; x_t | \mathbf{H}) = \sum_{t=1}^{T} I(\tilde{\gamma}_t).
$$

(4)

is given by the sum over the sub-channel mutual information which is a monotonically increasing function of the post equalization SINR on each sub-channel for a particular input distribution\(^1\) $p(x_t)$.

B. Channel Statistics and Channel Prediction

Let $\mathbf{H} \in \mathbb{C}^{T \times 1}$ denote the actual MIMO channel vector during the transmission of a particular block of interest, derived by stacking the columns of $\mathbf{H}$. Furthermore, let $\tilde{\mathbf{H}}_{\Delta}$ denote the CSI, available for adaptation $\Delta T_b$ seconds before the transmission of this frame, where $\Delta$ denotes the adaptation delay in number of blocks and $T_b$ is the block duration in seconds during which the channel remains static. More specifically, $\tilde{\mathbf{H}}_{\Delta}$ is composed of the $M$ MIMO channel vectors $[\tilde{\mathbf{H}}_{\Delta,1}, \ldots, \tilde{\mathbf{H}}_{\Delta,M}]^T$, where $\tilde{\mathbf{H}}_{\Delta,m}$ with $m = 1 \ldots M$ is delayed by $(\Delta + m - 1)$ blocks as compared to the actual block of interest. In the following it is assumed that $\mathbf{H}$ and $\tilde{\mathbf{H}}_{\Delta}$ are jointly complex Gaussian distributed with zero mean, auto covariance matrices $\mathbf{\Sigma}_{\mathbf{HH}}$ and $\mathbf{\Sigma}_{\tilde{\mathbf{H}}_{\Delta} \tilde{\mathbf{H}}_{\Delta}}$, and cross covariance matrix $\mathbf{\Sigma}_{\mathbf{H} \tilde{\mathbf{H}}_{\Delta}}$.

Using fundamental results for multivariate Gaussian random variables (e.g. [7, ch. 7.5]) the minimum mean square error channel predictor, which exploits spatial and temporal correlation follows as

$$
\bar{\mathbf{H}}_p = \mathbb{E}_{\tilde{\mathbf{H}}_{\Delta}} [\mathbf{H} | \tilde{\mathbf{H}}_{\Delta}] = \mathbf{\Sigma}_{\mathbf{HH}} \mathbf{\Sigma}_{\mathbf{HH}}^{-1} \mathbf{\Sigma}_{\mathbf{H} \tilde{\mathbf{H}}_{\Delta}} \tilde{\mathbf{H}}_{\Delta}.
$$

(5)

The parameter $M$ can be viewed as prediction filter length, which determines the prediction accuracy but also the computational complexity. Throughout the numerical examples, we assume that each channel coefficient has unit variance. For the sake of a small parameter space, we define the spatio-temporal correlation coefficient between each two channel coefficients at time instants $T_1$ and $T_2$ and receive-transmit antenna pairs $r_1, t_1$ and $r_2, t_2$ as

$$
\mathbb{E} [H_{T_1}^{r_1 t_1} H_{T_2}^{r_2 t_2}^*] = \rho_s (t_2 - t_1)^2 \rho_s (r_2 - r_1) \rho_s (T_2 - T_1).
$$

(6)

In (6) we assumed: 1) separability of transmitter and receiver spatial correlation with identical receiver and transmitter correlation coefficients $\rho_s$ (see [8]) and 2) separability of spatial and temporal correlation. The temporal correlation coefficient depends on the time difference and the maximum doppler frequency $f_{d,\text{max}}$ as $\rho_s (T_2 - T_1) = J_0(2\pi f_{d,\text{max}}(T_2 - T_1))$ where $J_0(\cdot)$ denotes the zeroth order Bessel function of the first kind (Jakes doppler spectrum [9, ch. 4.1]).

III. RATE ADAPTATION FOR LINEAR RECEIVERS

A. The Rate Adaptation Problem

In the following, we investigate the problem of how to compute the transmission rate $I'_t$ per sub-channel, given the CSI $\tilde{\mathbf{H}}_{\Delta}$, which is outdated from the perspective of the actual transmission. Thereby, we follow the argumentation in [10] and assume that the transmitted blocks are long enough such that the mutual information $I(\tilde{\gamma}_t)$ can be achieved. However, the transmitted blocks are short as compared to the channel coherence time, such that the block-static fading assumption holds. The only error event is hence the event of a mutual information outage, which occurs if the selected transmission rate exceeds the mutual information: $I'_t > I(\tilde{\gamma}_t)$. In that regard we pursue two tasks: either to maximize the mutual information with outage or to ensure that a certain target outage probability $P_{\text{out}}^t$ is achieved. Both tasks can be treated independently for each sub-channel.

The probability of outage is

$$
P_{\text{out}}(I'_t) = \Pr \{ I(\tilde{\gamma}_t) < I'_t | \tilde{\mathbf{H}}_{\Delta} \}
$$

(7)

$$
= \Pr \{ \tilde{\gamma}_t < \gamma'_t | \tilde{\mathbf{H}}_{\Delta} \} = \int_0^{\gamma'_t} p(\tilde{\gamma}_t | \tilde{\mathbf{H}}_{\Delta}) d\tilde{\gamma}_t.
$$

(8)

(8) states that an outage occurs equivalently to (7) if the post equalization SINR $\tilde{\gamma}_t$ during the future transmission is smaller than some predicted threshold value $\gamma'_t$ and the rate has been chosen as $I'_t = I(\tilde{\gamma}_t)$. This holds since the mutual information is monotonically increasing in $\tilde{\gamma}_t$. The mutual information with outage is thus

$$
I_{\text{out},t} = (1 - P_{\text{out}}(\tilde{\gamma}_t)) I(\tilde{\gamma}_t).
$$

(9)

The major difficulty in computing the outage probability in (8) is the lack of closed-form expressions for the conditional probability density function (pdf) $p(\tilde{\gamma}_t | \tilde{\mathbf{H}}_{\Delta})$, for arbitrary spatially correlated channels and arbitrary linear equalizers\(^2\).

\(^1\)The underlying assumption is that any residual post equalization interference is Gaussian distributed, which is only an approximation if $x$ is drawn from non-Gaussian or discrete alphabets. Each of the $T$ parallel sub-channels $x_t \rightarrow \tilde{x}_t$ is hence treated as an additive white Gaussian noise channel with SNR $\tilde{\gamma}_t$. See [6] for a detailed discussion.

\(^2\)Not even the marginal pdf $p(\tilde{\gamma}_t)$ is known in the arbitrary spatially correlated channel case or for MMSE based equalization (see [11] for approximations based on the Gamma distribution in case of MMSE based equalization). Considering the problem at hand, the actual channel of the future transmission, conditioned on the outdated channel $\tilde{\mathbf{H}}_{\Delta}$, has non zero mean, which further complicates deriving $p(\tilde{\gamma}_t | \tilde{\mathbf{H}}_{\Delta})$. 
To overcome this issue, we note that any function of the outdated CSI $\hat{\mathbf{H}}_\Delta$ can still be used to decrease the uncertainty about the future SINR to some extent\(^3\). A possible option is to use the post equalization SINR $\tilde{\gamma}_{t,p}$, computed from the predicted channel $\hat{\mathbf{H}}_p$ (see the MMSE predictor (5)), which is highly correlated to the actual SINR $\tilde{\gamma}_t$ if the outdated CSI $\hat{\mathbf{H}}_\Delta$ was. Again, the joint (bivariate) pdf of actual and predicted SINR $P(\tilde{\gamma}_t, \tilde{\gamma}_{t,p})$ is hard to obtain in closed form, since even its marginal distributions are not known in general. Therefore, we attempt to construct it from approximations to its marginal distributions $P(\tilde{\gamma}_t)$ and $P(\tilde{\gamma}_{t,p})$. More specifically, we consider approximations of the marginal distributions of the log-transformed SINR $\tilde{\gamma}_t = \log(\tilde{\gamma}_t)$ and $\tilde{\gamma}_{t,p} = \log(\tilde{\gamma}_{t,p})$ by the Gumbel and the Gaussian distribution, in order to construct $P(\tilde{\gamma}_t, \tilde{\gamma}_{t,p})$ or $P(\tilde{\gamma}_t | \tilde{\gamma}_{t,p})$ which is motivated in the following section. Similar to (8), the outage probability is given by

$$P_{\text{out}} (\tilde{\gamma}_t) = \int_{-\infty}^{\tilde{\gamma}_t} P(\tilde{\gamma}_t | \tilde{\gamma}_{t,p}) d\tilde{\gamma}_t$$

if the transmission rate has been chosen as\(^4\) $I_t' = 1(\exp(\tilde{\gamma}_t))$.

**B. Log-Transformed SINR Approximations**

Starting point for the approximation of the marginal distribution $P(\tilde{\gamma}_t)$ is the observation that the post equalization SINR with $R = T$ antennas, ZF based equalization and without spatial correlation follows an exponential distribution (see e.g. [12]): $P(\tilde{\gamma}_t) = 1/\gamma_t \exp(-1/\gamma_t) \log(\tilde{\gamma}_t)$ can hence be easily shown to follow an extreme value distribution [13], also known as Gumbel distribution:

$$P(\tilde{\gamma}_t) = 1/b_t \exp \left( z_{\tilde{\gamma}_t} - \exp(z_{\tilde{\gamma}_t}) \right)$$

(10)

with $z_{\tilde{\gamma}_t} = (\tilde{\gamma}_t - a_t)/b_t$. The shift and shape parameters $a_t$ and $b_t$ are related to average $\mu_{\tilde{\gamma}_t}$ and variance $\sigma^2_{\tilde{\gamma}_t}$ of $\tilde{\gamma}_t$ by [14]

$$b_t = \sigma_{\tilde{\gamma}_t}/\sqrt{6}, \quad a_t = \mu_{\tilde{\gamma}_t} - b_t u, \quad u : \text{Euler's constant}.$$ (11)

which could be exploited to estimate those parameters. For the zero forcing equalizer it is found that $a_t$ relates to the SNR as $a_t = \log(\gamma)$ and $b_t = 1$. However, we keep those parameters variable in order to adjust the distribution in case of spatially correlated channels or MMSE based equalization. The respective cumulative distribution function (cdf) is

$$F(\tilde{\gamma}_t) = 1 - \exp(-\exp(z)).$$ (12)

A numerical comparison of the log-transformed post equalization SINR distribution obtained from simulations with ZF and MMSE based linear equalization and uncorrelated as well as severely spatially correlated channels, is shown in Fig. 1. The Gumbel distribution matches the simulated SINR distribution perfectly in case of spatially uncorrelated channels and ZF equalization, as expected. However, the skewness of the SINR distribution turns out to be reduced in case of MMSE

\(^3\) Such a function would contain the same information about $\tilde{\gamma}_t$ as compared to $\hat{\mathbf{H}}_\Delta$, if it was a sufficient statistics with regard to $\tilde{\gamma}_t$.

\(^4\) The mutual information is a monotonically increasing function in $\tilde{\gamma}_t = \log(\tilde{\gamma}_t)$. An outage equivalently occurs in case $\tilde{\gamma}_t < \tilde{\gamma}_t^\circ$ but also if $\tilde{\gamma}_t^\circ < \tilde{\gamma}_t^\circ$.

Fig. 1: Post equalization log-SINR: Comparison of the Gumbel and the Gaussian approximation to the simulated distribution in a $2 \times 2$ MIMO system

Based equalization or severely correlated channels such that a representation by the Gumbel distribution becomes inaccurate. However, the observed reduction in skewness motivates a second approximation of the log-transformed post equalization SINR, namely an approximation by the Gaussian distribution, matched to mean and variance of $\tilde{\gamma}_t$, which is also shown in Fig. 1.

In the following we attempt to construct the bivariate joint distribution of $\tilde{\gamma}_t$ and $\tilde{\gamma}_{t,p}$ from its marginal distributions. It should be noted, however, that in principle there exists infinitely many solutions for a joint distribution with given marginal distributions as already pointed out in [15] and earlier work\(^5\).

1) **Bivariate Gumbel Distribution**: [17] studied several ways on how to construct a bivariate distribution from given marginals. The following form appears particularly well suited\(^6\)

$$F(\tilde{\gamma}_t, \tilde{\gamma}_{t,p}) = \exp \left( - \left[ -\log(F(\tilde{\gamma}_t)) \right]^m + \left[ -\log(F(\tilde{\gamma}_{t,p})) \right]^m \right)^{1/m}$$

(13)

with marginal cdfs according to (12). The parameter $m \geq 1$ characterizes the statistical dependency between $\tilde{\gamma}_t$ and $\tilde{\gamma}_{t,p}$ and is related to their temporal correlation coefficient $\rho_t$ through $m = (1 - \rho_t)^{-1/2}$ [16, ch. 53]. For $m = 1$, corresponding to $\rho_t = 0$, (13) decomposes into a product of the marginal cumulative distribution functions.

The outage probability can be obtained from the joint distribution in (13) by using general results for distribution

\(^5\) Generally, constructing a bivariate distribution requires to know its marginal distributions and in addition a dependence function [16, ch. 19], describing the coupling between both random variables.

\(^6\) The correlation coefficient $\rho_t$ between predicted and actual SINR has to satisfy $0 \leq \rho_t \leq 1$. Other options have been studied in [17] as well, where the range of the correlation coefficient is restricted to be smaller.
functions (e.g. [18, ch.2, (2.1-32)])

\[ P_{\text{out}}(\tilde{\gamma}_{l,t}^t) = \frac{\partial F(\tilde{\gamma}_{l,t}^t, z_{l,t}^t)}{\partial z_{l,t}^t} \frac{1}{p(\tilde{\gamma}_{l,t}^t)}. \]  

(14)

By calculating the derivative in (14), the outage probability can be stated in closed form as follows

\[ P_{\text{out}}(\tilde{\gamma}_{l,t}^t) = \exp \left( - \left( c_{l,t}^m + c_{l,t}^m \right) \frac{\tilde{\gamma}_{l,t}^t}{a_{l,t}^m} \right) \frac{1}{1 - \exp \left( -\exp \left( z_{l,t}^t \right) \right)}. \]

(15)

with

\[ c_{l,t}^m = - \log \left( 1 - \exp \left( -\exp \left( z_{l,t}^t \right) \right) \right), \quad z_{l,t}^t = \frac{c_{l,t}^m}{b_{l,t}^m} \]

\[ c_{l,t}^m = - \log \left( 1 - \exp \left( -\exp \left( z_{l,t}^t \right) \right) \right), \quad z_{l,t}^t = \frac{c_{l,t}^m}{b_{l,t}^m}. \]

(15)

is hence fully specified by the temporal correlation coefficient \( \rho_t \) and the shift and shape parameters \( a_t, b_t \) and \( a_{l,t}, b_{l,t} \) which are derived from the (joint) first and second order moments of \( \tilde{\gamma}_{l,t}^t \) and \( \tilde{\gamma}_{l,t}^t \) using (11).

2) Bivariate Gaussian Distribution: Based on the observation that the log-transformed post equalization SINR approximately follows a Gaussian distribution if MMSE based equalization is employed or the channel becomes spatially correlated, we assume that \( \tilde{\gamma}_{l,t}^t \) and \( \tilde{\gamma}_{l,t}^t \), used for the fundamental results for Gaussian random variables (see e.g. [7, ch. 7.5]) as follows

\[ \mu_t = \mu_{l,t}^t + \sigma_t^2 \tilde{\gamma}_{l,t}^t - \mu_{l,t}^t \]

(16)

\[ \sigma_t^2 = \tilde{\gamma}_{l,t}^t - \sigma_{l,t}^2 \tilde{\gamma}_{l,t}^t \]

(17)

The outage probability can then be obtained from \( p(\tilde{\gamma}_{l,t}^t | \tilde{\gamma}_{l,t}^t) \) as solution to the integral

\[ P_{\text{out}}(\tilde{\gamma}_{l,t}^t) = \int_{-\infty}^{\tilde{\gamma}_{l,t}^t} p(\tilde{\gamma}_{l,t}^t) d\tilde{\gamma}_{l,t}^t = 1 - \frac{1}{2} \text{erfc} \left( \frac{\tilde{\gamma}_{l,t}^t - \mu_t}{\sqrt{2\sigma_t^2}} \right), \]

(18)

where \( \text{erfc}(u) = \int_u^{\infty} z/\sqrt{\pi} \exp(-t^2)dt \) denotes the complementary error function. (18) provides a remarkably simple means to evaluate the outage probability approximately and hence to adjust \( \tilde{\gamma}_{l,t}^t \), obtained from the predicted channel as

\[ \tilde{\gamma}_{l,t}^t = \text{erfc}^{-1} \left( 2 - 2P_{\text{out}} \right) \sqrt{2\sigma_t^2} + \mu_t. \]

(19)

In (19) b) is the expected value \( E[\tilde{\gamma}_{l,t}^t | \tilde{\gamma}_{l,t}^t, \tilde{\gamma}_{l,t}^t] \), solely depending on the predicted log-transformed SINR through (16). a) is a constant correction term, which does only depend on the variance \( \sigma_t^2 \) of the conditional pdf, which characterizes the uncertainty contained in \( \tilde{\gamma}_{l,t}^t \), about \( \tilde{\gamma}_{l,t}^t \), and the desired target outage probability \( P_{\text{out}} \). Obviously, increasing the uncertainty \( \sigma_t^2 \) e.g. through higher velocities or decreasing \( P_{\text{out}} \) results in choosing \( \tilde{\gamma}_{l,t}^t \) more conservatively. (19) could be implemented by means of a lookup table of \( \text{erfc}^{-1} \).

C. Recursive Parameter Estimation

Both approximations require knowledge of the joint first and second order moments of \( \tilde{\gamma}_{l,t}^t \) and \( \tilde{\gamma}_{l,t}^t \) which are not known a priory but could be estimated recursively during the transmission of multiple frames as follows. Let \( a[i] \) and \( b[i] \) be the i-th realization of a wide sense stationary random process. Estimates of mean and covariance (likewise for the variance) are obtained from the recursive mean estimator

\[ \tilde{\mu}_a[i] = \frac{i-1}{i} \tilde{\mu}_a[i-1] + \frac{1}{i} a[i] \]

\[ \tilde{\sigma}_{ab}[i] = \frac{i-2}{i-1} \tilde{\sigma}_{ab}[i-1] + \frac{1}{i} (a[i] - \tilde{\mu}_a[i]) (b[i] - \tilde{\mu}_b[i]). \]

Note that \( \tilde{\gamma}_{l,t}^t \) is available \( \Delta \) blocks after the \( \tilde{\gamma}_{l,t}^t \) such that the moments cannot be estimated during the first frames of a transmission. This could be circumvented by starting the estimation a number of frames before the actual transmission, which also improves the estimation accuracy.

IV. NUMERICAL EXAMPLES

A 2 x 2 MIMO system operating at 2.5 GHz is investigated, where MIMO channel vectors \( \tilde{H}_\Delta \) from \( M = 2 \) blocks, with a delay of \( \Delta = 8 \) blocks (\( T_b = 8 \) ms) are available during adaptation. Firstly, the SINR \( \tilde{\gamma}_{l,t}^t \) and hence the rate which achieves a target outage probability \( P_{\text{out}} = 0.1 \) is computed from the outage probability (15) or (18) using a bisection algorithm. The respective results are denoted \( \text{Gumbel} \) and \( \text{Gauss} \), depending on the used approximation. Secondly, the mutual information with outage in (9) is maximized using a combination of bisection and direct search algorithm [19]. The respective results are denoted \( \text{Gumbel max} \) and \( \text{Gauss max} \). For comparison we consider a straight forward adaptation strategy, where the post equalization SINR \( \tilde{\gamma}_{l,t}^t \), computed from the MMSE channel prediction (5), is used to select the transmission rate as \( I'_t = I(\tilde{\gamma}_{l,t}^t) \) (denoted \( \text{MMSE} \)). Results at different velocities and for ZF and MMSE based equalization, with and without spatial correlation are shown in Fig. 2.

The \( \text{MMSE} \) adaptation causes the outage probability and the mutual information with outage to decrease with increasing velocity. This can be explained from the fact that the channel prediction becomes increasingly biased with increasing velocity\(^7\). In particular, the outage probability is about 0.5 in the low velocity regime, which highlights the need for a reliable rate adaptation. The non-monotonic shape of the curves is due to the temporal correlation function of the channel which follows a Bessel function. It can be seen that the \( \text{Gumbel} \) and \( \text{Gauss} \) approximations can guarantee not to exceed the \textit{outage probability target}. However, the rate is chosen too conservative in the low velocity regime, which indicates that the construction of the bivariate distribution \( p(\tilde{\gamma}_{l,t}^t | \tilde{\gamma}_{l,t}^t) \) not completely captures the high statistical dependency between \( \tilde{\gamma}_{l,t}^t \) and \( \tilde{\gamma}_{l,t}^t \). In case of MMSE based equalization, the Gaussian approximation turns out to be more accurate,
due to the reduced skewness of the log-SINR distribution. The gain of exploiting the statistical dependency between $H_\Delta$ and $\mathbf{H}$ becomes obvious as the mutual information with outage significantly increases in the low velocity regime at a constant outage probability. It almost approaches the mutual information with perfect adaptation (no outages). However, higher velocities and hence an increased uncertainty about the channel during the actual transmission\footnote{The pdf $p(\tilde{\gamma}_l^\star | \gamma_l^\star, \rho)$ becomes increasingly spread with increasing velocities.} cause a large mutual information loss compared to the case of perfect CSI, since the uncertainty about the future channel quality cannot be reduced by the available CSI during adaptation. Note that a higher prediction filter length $M$ (not shown) would increase the velocity range, where the velocity caused loss remains small.

Regarding the \textit{maximal mutual information with outage} it can be seen, that small outage probabilities are optimal in the low velocity regime, whereas an outage probability of about 0.4, i.e., a less conservative transmission becomes optimal at high velocities. As expected, the mutual information of the MMSE based adaptation is exceeded over the complete velocity range. Again, the Gaussian approximation turns out to be more accurate considering MMSE based equalization and spatial correlation as compared to the Gumbel distribution and hence achieves a higher mutual information with outage.

V. CONCLUSIONS

Rate adaptation for MIMO spatial multiplexing receivers based on outdated CSI has been investigated in this paper. It was shown that the log-transformed post equalization SINR distribution can be approximated by the Gumbel distribution and, more accurately, by the Gaussian distribution. These approximations serve as basic tools to express the outage probability in closed form. Those expressions are suited to adjust the transmission rate adaptively on a per block basis in order to achieve a target outage probability or to maximize the mutual information with outage. Open issues which are deferred to future work comprise to incorporate channel estimation noise, the extension to a transmission over multiple independently fading frequency bands in broadband multi-carrier systems, the application to successive interference cancelation receivers and finally the combination with higher layer error correction mechanisms such as hybrid ARQ.

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