RESEARCH ARTICLE

Digital compensation of transmitter leakage in FDD zero-IF receivers

A. Frotzscher\textsuperscript{1*} and G. Fettweis\textsuperscript{2}

\textsuperscript{1} Bell LabsAlcatel-Lucent, Stuttgart, Germany
\textsuperscript{2} Vodafone Chair Mobile Communications SystemsTechnische Universität Dresden, Dresden, Germany

ABSTRACT

Transmitter leakage (TxL) has a significant impact on the performance of frequency division duplex devices using zero-IF receivers and thus requires a suitable compensation. In contrast to analog approaches, digital TxL compensation approaches can easily be reconfigured and therefore are highly attractive for multiband mobile devices. In this contribution, we present a system model, describing the TxL impact on the received digital baseband signal. Furthermore, we show analytically that TxL causes an additional, irreversible signal-to-noise loss due to the quantisation in the analog-to-digital converter, which limits the applicability of a digital TxL compensation. However, this signal-to-noise ratio loss remains negligibly small as long as the TxL interference does not exceed significantly the desired received signal. For TxL to be compensated digitally, the TxL channel needs to be estimated. In this work, we present two TxL channel estimation approaches suitable for frequency flat and frequency selective channels. It is shown that the usage of these approaches mitigates effectively the TxL impact on the system performance. Thus, the system designer can allow much stronger TxL, which significantly relaxes the requirements on the transmitter–receiver isolation of the duplexer and the linearity of the analog receiver frontend. Copyright © 2011 John Wiley & Sons, Ltd.

*Correspondence
Andreas Frotzscher, Bell Labs, Alcatel-Lucent, Lorenzstr. 10, Stuttgart, Germany.
E-mail: andreas.frotzscher@alcatel-lucent.com

Received 19 January 2011; Revised 10 August 2011; Accepted 29 August 2011

1. INTRODUCTION

In compact wireless transceivers (e.g. handset devices or small cell base stations) operating in the frequency division duplex (FDD) mode, a duplexer is used to connect the transmitter (Tx) and receiver (Rx) chain with a common antenna as well as to isolate the Rx chain from the powerful transmit signal. The continuing miniaturisation of the transceiver hardware complicates achieving a sufficient Tx–Rx isolation. This problem becomes even worst by the demand to support several wireless communication standards with different carrier frequencies and bandwidths with one single transceiver. Keeping the complexity of the hardware as low as possible motivates the development of a software defined radio [1], that is a reconfigurable analog frontend being digitally tunable to different carrier frequencies and bandwidths with one single transceiver. The continuing miniaturisation of the transceiver hardware complicates achieving a sufficient Tx–Rx isolation. This problem becomes even worst by the demand to support several wireless communication standards with different carrier frequencies and bandwidths with one single transceiver. Keeping the complexity of the hardware as low as possible motivates the development of a software defined radio [1], that is a reconfigurable analog frontend being digitally tunable to different carrier frequencies and bandwidths. Modern transceivers have to support high data rates, which sets challenging requirements on the analog hardware (e.g. linearity, noise figure). However, the design of a frequency agile analog frontend complicates the fulfillment of these requirements. Therefore, the duplexers Tx–Rx isolation can become insufficient, and thus, a significant part of the transmit signal leaks into the Rx chain and can deteriorate severely the demodulation of the received signal [2]. Because the zero-infrared (zero-IF) receiver architecture offers various advantages for the design of a frequency agile analog frontend [1], the present work focuses on transmitter leakage (TxL) in this Rx architecture.

The practical design of a zero-IF receiver has to face several implementation problems [1] (e.g. in-phase/quadrature-phase (I/Q) imbalance, phase noise, direct current (DC) offset and second-order intermodulation products). Additionally, in FDD zero-IF receivers, the nonlinearity of the down converter generates a second-order intermodulation product (IM2) of the leaking transmit signal, which is located around DC and thus interfere the desired, down converted received signal [3]. Although the leaking transmit signal causes other additional interferences, this IM2 can be considered as the dominating TxL interference [4].

The problem of TxL is conventionally resolved by placing an additional band pass filter at the input or output of the low noise amplifier to suppress the leaking transmit
For a sufficient TxL suppression, surface acoustic wave filters are employed usually. However, because these filters cannot be reconfigured and integrated monolithically, they are not attractive for frequency agile frontends. Several authors proposed two other TxL mitigation approaches using an adaptive filter in parallel to the duplexer [6] or in front of the down converter [7] and provide a suppression of the leaking transmit signal of approximately 20 dB. Nevertheless, operating in the analog radio frequency stage, these approaches increase significantly the complexity of the analog frontend and have a limited reconfigurability. Therefore, they are not appealing for frequency agile frontends. In contrast to this, compensating the TxL interference in the digital baseband has two major advantages. First, the digital TxL compensation relaxes the requirements on the analog frontend, that is Tx–Rx isolation of the duplexer and the linearity of the Rx frontend. Second, it is independent of the supported carrier frequency and can be reconfigured easily to different signal bandwidths. Whereas the analog compensation of TxL is already well known in the literature, the digital TxL compensation is a new emerging approach and was proposed independently by [8] and [9] for the first time.

However, digital TxL compensation approaches force the analog-to-digital converter (ADC) in the Rx chain to cope with the TxL. Consequently, a part of the ADC quantisation resolution is spent inherently for digitising the TxL interference and yields an additional, irreversible signal-to-noise ratio (SNR) loss with respect to the desired, received signal. Therefore, this work derives analytically the SNR loss due to TxL and, by this, presents a lower bound of the applicability of the digital TxL compensation. However, it will be shown that this SNR loss remains negligibly small as long as the TxL interference does not significantly exceed the desired received signal. Thus, compensating the TxL interference in the digital domain is highly attractive for low cost zero-IF receivers, and beyond this, it facilitates significantly the design of a frequency agile analog frontend.

For the TxL interference to be compensated digitally, the TxL channel must be estimated, which represents the combined filtering of the leaking transmit signal by the Tx chain, the Tx–Rx isolation of the duplexer and the Rx chain. In this work, we present two approaches for the estimation of the TxL channel. The first one is based on a stochastic gradient least mean squares (LMS) estimator and estimates the mean attenuation of the TxL channel. It has a very low complexity, but its estimation performance deteriorates severely for frequency selective TxL channels. In contrast to this, the second presented approach, using a least squares (LS) estimator, is suitable for frequency selective TxL channels in general.

The outline of the paper is as follows. In Section 2, the baseband system model is given. Section 3 analyses the impact of the TxL interference on the ADC in the Rx chain. The two digital TxL estimation approaches are presented in Section 4, and their performance are compared in Section 5. Conclusions are drawn in Section 6.

2. SYSTEM MODEL

In favour of a comprehensible signal notation, this work focuses on the TxL present in the mobile handsets. Nevertheless, the presented results can be translated easily to the TxL present at base stations. The block diagram of such a zero-IF transceiver is depicted in Figure 1. The modulator generates the discrete, oversampled uplink (UL) signal $s_{UL}[k]$, which is converted to analog domain. The resulting analog signal $s_{UL}(t)$ is up-converted to the UL carrier frequency, amplified by the power amplifier and leaks partly through the duplexer into the Rx chain as consequence of the insufficient Tx–Rx isolation. The leaking UL signal is amplified and filtered between the digital-to-analog converter (DAC) in the Tx chain and the I/Q down converter in the Rx chain. This combined amplification and filtering can be considered as the effective TxL channel. The I/Q down converter in the Rx chain converts the received desired downlink (DL) signal to baseband. However, it also
generates a second-order intermodulation product (IM2) of
the leaking UL signal, which interfere the down converted
DL signal and is considered as the dominating TxL interfer-
ence [4]. The down converter is followed by the channel
select filters (CSF) in the I-branch and Q-branch and a
DC offset compensation (DC−1). In order to adapt to the
dynamic range of the ADC, the signal is amplified with a
variable gain amplifier (VGA), being part of an automatic
gain control.

It is important to note that usually both the DAC in
the Tx chain and the ADC in the Rx chain employ an
oversampling due to the digital impulse shaping and time
synchronisation purpose. The employed ADC and DAC
oversampling factor ? ranges typically between 2 and 8.

As will be seen later, the TxL interference may have
unequal signal power in the I-branch and Q-branch,
demanding an unequal I/Q VGA amplification. For an
increase of the I/Q imbalance distortion of the received
DL signal to be prevented, the VGA I/Q gain imbalance is
corrected after the ADC. This yields the discrete baseband
signal r[k], which can be modelled as

\[ r[k] = (h_{DL} * s_{DL})[k] + w[k] \]
\[ + c h_1[k] + j c_q h_Q[k] + h_{UL} \times s_{UL}[2][k] + \epsilon \]

(1)

The term a[k] in Equation (1) describes the received
DL signal, arising from the convolution of the ?-times
oversampled, discrete DL signal s_{DL}[k] with the discrete
impulse response h_{DL}[k], which represents the DL trans-
mition channel and the CSF. The phase noise and the
I/Q imbalance impact of the Rx chain on the DL signal is
excluded from the equations because it does not affect
the TxL estimation in the digital baseband.

The term b[k] in Equation (1) describes the DC-free TxL
interference. It was first derived in [3], assuming identi-
cal CSF in the I-branch and Q-branch with a passband as
wide as the TxL interference. The ?-times oversampled,
discrete UL signal s_{UL}[k] is distorted by the TxL channel,
represented by the equivalent discrete impulse response
h_{TxL}[k] with the channel length L_{TxL}. For comprehensi-
ble and reproducible results to be provided, a model of the
TxL channel impulse response is required. Because no
TxL channel models are available in the literature so far,
a heuristic channel model was found on the basis of measure-
ment results of a surface acoustic wave duplexer, presented
in [10]. This TxL channel model is assumed throughout
this work.

\[ h_{TxL}[k] = \begin{cases} c_{TxL} e^{-\tau} e^{j \varphi_{TxL}[k]} & k = 0 \\ c_{TxL} e^{-\frac{k}{2 \tau}} e^{j \varphi_{TxL}[k]} & 1 \leq k \leq L_{TxL} - 1 \end{cases} \]

(2)

The tap magnitude is deterministic according to the
decay constant \( \tau \), whereas the tap phases \( \varphi_{TxL}[k] \) are inde-
pendent, uniformly distributed within \([0, 2\pi]\). The scaling
factor \( c_{TxL} \) is adjusted according to the desired TxL
interference condition.

The I/Q down converter in the Rx chain consists of a
mixer in the I-branch and Q-branch. As consequence
of substrate coupling, self mixing and a limited linearity
and symmetry of the circuitry, the mixers generate
second-order intermodulation products (IM2), which are
represented at the mixer output by a signal term propor-
tional to the square of the radio frequency input signal
[11]. At baseband, the IM2 of the leaking UL signal is
represented by the magnitude square of the convolution
product \( (h_{UL} * s_{UL})[k] \). Because the nonlinearity of the
mixers results from unavoidable production tolerances of
the mixer circuitry, the IM2 in the I-branch and Q-branch
differ from each other by a random, time-invariant I/Q mis-
match factor, denoted by c_q. The factor c in Equation (1)
is a sign variable \( c \in \{+1, -1\} \) and describes the sign
of the mixer nonlinearity. The impulse responses of the
CSF in the I-branch and Q-branch are denoted by h_{I}[k]
and h_{Q}[k], respectively. The DC offset compensation intro-
duces a counter phased signal to mitigate the TxL DC off-
sit and other static or dynamic DC offsets [12]. However,
because the TxL channel exhibits a higher time variance
[10], the DC offset compensation can be represented in
Equation (1) by the constant \( \epsilon \). Finally, \( w[k] \) is a complex
noise signal, representing the thermal noise and quantisa-
tion noise and shall be modelled as zero-mean Gaussian
noise.

For compensating digitally the TxL interference, the
parameters of b[k] need to be estimated on the basis of the
observation of the received baseband signal r[k] and UL
signal s_{UL}[k]. Using these estimates, the TxL interference
is reconstructed and subtracted as the compensation signal
s_C[k] from r[k]. For performance evaluation, let \( G_C \) denote
the compensation gain, excluding DC, defined as

\[ G_C = \frac{E_k \{ \lvert b[k]\rvert^2 \}}{E_k \{ \lvert b[k] - s_C[k]\rvert^2 \}} \]

(3)

It describes the reduction of the TxL interference power
by the digital TxL compensation. Furthermore, let \( \gamma_{TxL} \)
denote the TxL signal-to-interference ratio (SIR) between
the DL signal and the TxL interference, present in r[k],
within the DL signal bandwidth BW

\[ \gamma_{TxL} = \frac{E \{ \lvert a[k]\rvert^2 \}}{g_b E \{ \lvert b[k]\rvert^2 \}} \]

(4)

The factor g_b describes the ratio of the TxL interference
power within BW to the total interference power. Similarly,
\( \gamma \) denotes the SNR of the DL signal and is defined as

\[ \gamma = \frac{E \{ \lvert a[k]\rvert^2 \}}{g_w E \{ \lvert w[k]\rvert^2 \}} \]

(5)
where $g_w$ describes the ratio between the power of noise signal $w[k]$ within $BW$ and the total signal power.

### 3. Transmitter Leakage Impact on the Analog-to-Digital Conversion

Compensating the TxL interference in the digital band implicates the analog-to-digital conversion of the composite signal, containing the received DL signal, the TxL interference and the thermal noise. Consequently, a fraction of the ADC quantisation resolution is spent for the TxL interference. This causes an additional, irreversible SNR loss to the received DL signal and thus limits the applicability of the digital TxL compensation. The impact of TxL on the SNR at the digital baseband was first analysed in [13] and will be reviewed here.

The block diagram of the AD conversion unit in the Rx chain is depicted in Figure 2. The VGA input signal can be expressed similar to Equation 1 by

$$r^{(VGA)}(t) = \left( h_{DL}\ast s_{DL}\right)(t) + \hat{w}(t) + \tilde{a}(t) + c(h_I + j c q h_Q)(t)\ast \left| h_{TxL}\ast s_{UL}\right|^2(t) + \tilde{c}$$

where $\tilde{a}(t)$, $\tilde{b}(t)$ and $\tilde{w}(t)$ describes the received DL signal, the TxL interference and the thermal noise signal at the VGA input, respectively. The DL signal $\hat{a}(t)$, the TxL interference $\hat{b}(t)$ and the thermal noise signal $\tilde{w}(t)$ can be considered as independent random processes. Thus, the variance of the VGA input signal equals the sum of the variances of the three signals (i.e. $\sigma^2_{r^{(VGA)}} = \sigma^2_a + \sigma^2_b + \sigma^2_{\tilde{w}}$). The conversion gain of the down converter and gain of the subsequent components differ slightly in the I-branch and Q-branch. However, this I/Q gain imbalance is typically only a fraction of a decibel [14]. Thus, the real and imaginary parts of the received DL signal have approximately the variance $\sigma^2_a/2$. Similarly, the real and imaginary parts of the noise signal can be interpreted as two-independent, real-valued processes with the variance $\sigma^2_{\tilde{w}}/2$, resulting from the band limitation of white Gaussian processes by the CSF. Depending on the CSF transfer function, $\tilde{w}(t)$ may contain spectral components outside the DL signal bandwidth $BW$. Because only the spectral components within $BW$ disturb the demodulation of the DL signal, the SNR at the VGA input is given by

$$\gamma = \frac{\sigma^2_a}{g_w \sigma^2_{\tilde{w}}}$$

where $g_w$ describes the power ratio of $\tilde{w}(t)$ within $BW$ to the total noise power. In contrast to the DL signal and noise signal, the variance of the TxL interference $\sigma^2_{\tilde{w}}$ may be split unequally on the I-branch and Q-branch, according to the TxL I/Q mismatch $c_q$:

$$\sigma^2_{\tilde{w},I} = \frac{1}{1 + c_q} \sigma^2_{\tilde{w}}, \quad \sigma^2_{\tilde{w},Q} = \frac{c_q}{1 + c_q} \sigma^2_{\tilde{w}}$$

As shown in [2] and [3], the TxL interference $\hat{b}(t)$ has twice the UL signal bandwidth, and its power spectral density (PSD) depends on the transmission scheme (i.e. single carrier or multiple carrier). Similar to the noise signal, the TxL interference is band-limited by the CSF, and its interference power within $BW$ can be expressed by $g_w \sigma^2_{\tilde{w}}$, where $g_w \in [0, 1]$ depends on the TxL PSD and the CSF transfer function.

The VGA gain in the I-branch and Q-branch is adjusted periodically such that the real and imaginary parts of the ADC input signal exhibit a predefined signal variance $\sigma^2_a/2$ according to the ADC dynamic range. Considering the steady state of the VGA gain adjustment, the VGA gain factors in the I-branch and Q-branch are defined by

$$X_I = \frac{\sigma^2_a}{\sqrt{\sigma^2_a + \sigma^2_{\tilde{w}} + \frac{2}{1 + c_q} \sigma^2_{\tilde{w}}}}$$

$$X_Q = \frac{\sigma^2_a}{\sqrt{\sigma^2_a + \sigma^2_{\tilde{w}} + \frac{2c_q}{1 + c_q} \sigma^2_{\tilde{w}}}}$$

As can be seen, the TxL I/Q mismatch $c_q$ provokes an I/Q mismatch of the VGA gains. For an increase of the

---

**Figure 2.** Block diagram of the analog-to-digital conversion unit in the receiver chain.
I/Q imbalance distortion of the DL signal to be prevented, this VGA I/Q mismatch \( f_q = \frac{x_0}{x_1} \) should be corrected after the ADC, as already shown in Figure 2. Sampling \( r^{\text{ADC}}(t) \) with the sampling period \( T_s = 1/(\eta BW) \) yields the time discrete signal

\[
x[k] = x_1[k] + jx_0[k] = r^{\text{ADC}}(kT_s)
\]

### 3.1. Statistical properties of the sampled receiver signal

For the SNR after the ADC quantisation and VGA I/Q mismatch correction to be derived, the probability density function (PDF) of \( x_1[k] \) and \( x_0[k] \) must be derived. For this purpose, two transmission scenarios are considered.

- Scenario 1: DL - orthogonal frequency division multiplex (OFDM), UL - single carrier
- Scenario 2: DL - OFDM, UL - OFDM

Let us consider the scenario 1 first. It is well known that the real and imaginary parts of an OFDM time domain signal with a sufficient number of independent subcarriers can be well approximated by two independent, zero-mean Gaussian processes, having the same variance. The DL channel and the CSF only affect the variance and correlation properties of the DL signal, not its Gaussian characteristic. The impulse shaped single-carrier UL signal does not exhibit a Gaussian distribution, as was shown numerically by Aparin in [15]. Obviously, the TxL channel, the nonlinearity of the Rx chain and the CSF do affect the PDF of the TxL interference. However, to the authors best knowledge, this impact has not been analysed so far. Nevertheless, because the received DL signal, thermal noise and the TxL interference are independent of each other, the PDF of the quantiser input signal is given by convolving the PDFs of these three signals, which converges to a Gaussian distribution at high TxL SIR \( \gamma_{TxL} \). This motivates the approximation of the real and imaginary parts of the quantiser input signal as zero-mean Gaussian processes with equal variance \( \sigma_G^2 \). The suitability of this approximation will be shown later. Thus, the PDF of \( x_1[k] \) and \( x_0[k] \) is well approximated by

\[
f_{x_1}(x), f_{x_0}(x) \approx \frac{1}{\sqrt{\pi \sigma_x^2}} e^{-\frac{x^2}{\sigma_x^2}} \tag{9}
\]

Let us now consider the transmission scenario 2, considering an OFDM transmission in the DL and UL. As already explained in the scenario 1, the combination of the received DL signal and the thermal noise can be approximated by a correlated, zero-mean, complex Gaussian process, whose real and imaginary parts are independent and have the variances

\[
\sigma_{G,1}^2 = \frac{\sigma_G^2(1 + c_q^2)(\sigma_D^2 + \sigma_W^2)}{2(1 + c_q^2)2\sigma_D^2 + \sigma_W^2 + 4\sigma_D^2} \tag{10}
\]

\[
\sigma_{G,0}^2 = \frac{\sigma_G^2(1 + c_q^2)(\sigma_D^2 + \sigma_W^2)}{2(1 + c_q^2)2\sigma_D^2 + \sigma_W^2 + 4\sigma_D^2} \tag{11}
\]

Similar to the received DL signal, the leaking UL signal at the input of the I/Q down converter can also be modelled by a zero-mean, complex Gaussian process with independent real and imaginary parts. As well known, the magnitude square of a complex Gaussian process with independent real and imaginary parts exhibits an exponential distribution. Therefore, the PDF of the TxL interference at baseband in the I-branch and Q-branch can be approximated by two dependent exponential distributed processes, differing only by the TxL I/Q mismatch factor \( c_q \). The CSF slightly smooth the TxL PDF, and the DC offset compensation suppresses the mean of the distribution. Consequently, the quantiser input signals \( x_1[k] \) and \( x_0[k] \) can be approximated by the sum of a zero-mean Gaussian process, representing the received DL signal and the thermal noise, and a zero-mean exponential process, modelling the TxL interference. The variances of the exponential processes in the I-branch and Q-branch are given by

\[
\sigma_{E,1}^2 = \frac{\sigma_G^2 x^2}{(1 + c_q^2)(\sigma_D^2 + \sigma_W^2) + 2\sigma_D^2} \tag{12}
\]

\[
\sigma_{E,0}^2 = \frac{\sigma_G^2 x^2}{(1 + c_q^2)(\sigma_D^2 + \sigma_W^2) + 2\sigma_D^2} \tag{13}
\]

Consequently, the PDF \( f_{x_1}(x) \) of the quantiser input signal \( x_1[k] \) results from the convolution of a zero-mean Gaussian distribution with the variance \( \sigma_{G,1}^2 \) and a zero-mean exponential distribution with the variance \( \sigma_{E,1}^2 \). After some transformations, \( f_{x_1}(x) \) can be written as

\[
f_{x_1}(x) = \exp \left( \frac{\sigma_{G,1}^2 - 2\sigma_{E,1}(x + \sigma_{E,1})}{2\sigma_{E,1}^2} \right) \cdot \\
1 + \frac{1}{4}(x + \sqrt{x^2 - 2\sigma_{G,1}}) \cdot \frac{\sigma_{G,1}}{\sqrt{2}\sigma_{E,1}} \tag{14}
\]

where \( \text{erf}(x) = (2\sqrt{\pi}) \int_0^x e^{-t^2} dt \) describes the error function. The PDF \( f_{x_0}(x) \) can be calculated similar to Equation (14).

For the presented signal models to be validated, they are compared with the distributions of the quantiser input signal \( x_1[k] \). For this purpose, scenario 1 considers a 16-QAM single-carrier UL transmission. In the DL, an OFDM transmission similar to the Long Term Evolution (LTE) standard 2.5 MHz mode (256 subcarriers) with 126 data subcarriers (16-QAM) and 22 pilot subcarriers (binary phase shift keying) is assumed. The second scenario employs the described LTE similar to OFDM transmission in the UL and DL. Furthermore, a six-tap TxL channel is assumed according to Equation (2) with \( \tau = 1 \). The DL channel is...
modelled according to the International Telecommunication Union Vehicular A channel model with block fading and a fourth-order Chebyshev low-pass filter is used as CSF.

Figure 3 opposes the numerically calculated distributions of the quantiser input signal \( x_1[k] \) of scenarios 1 and 2 with the analytical PDF models (Equations (9) and (14)) in a normal probability plot for the case of a dominant TXL interference \( \gamma_{\text{TXL}} = -10 \text{ dB} \).

![Figure 3. Normal probability plot of \( x_1 \) of scenarios 1 and 2](image)

As the figure shows, the analytical PDF models match in over 80% of the probability mass with the numerically calculated distributions, which validates the analytical description of the quantiser input signals.

### 3.2. Signal-to-distortion-and-quantisation noise ratio

The SNR after the digital VGA I/Q mismatch correction is derived according to the approach presented by Dardari in [16]. Because the analog-to-digital conversion in the I-branch and Q-branch is carried separately and similarly, only the quantisation noise effects in the I-branch are analysed first. The quantisation can be described by a staircase function \( Q(\cdot) \) with \( 2^B \) output levels in the interval \([-\mathcal{T}, \mathcal{T}]\), where \( B \) denotes the bit resolution of the ADC. As mentioned before, the VGA gain is adjusted such that \( x_1[k] \) reaches the signal power \( \sigma_{x_1}^2 = \sigma_{\mathcal{x}}^2 / 2 \) and ensures the predefined clipping ratio \( \rho = \mathcal{T} / \sigma_x \). Because \( Q(\cdot) \) is a static nonlinearity and \( x_1[k] \) can be considered as a zero-mean process, the output signal \( y_1[k] \) can be interpreted as the sum of a scaled version of \( x_1[k] \) and a distortion noise \( d_1[k] \)

\[
y_1[k] = Q(x_1[k]) = \kappa_1 x_1[k] + d_1[k] \quad (15)
\]

where the distortion noise \( d_1[k] \) is uncorrelated with the input signal \( x_1[k] \), and thus, the scaling factor \( \kappa_1 \) is defined by the ratio of the input–output cross correlation function of \( x_1[k] \) and \( y_1[k] \) and the auto correlation function of \( x_1[k] \), both at a delay time 0.

\[
\kappa_1 = \frac{E[x_1[k]y_1[k]]}{\sigma_{x_1}^2} = \frac{1}{\sigma_{x_1}^2} \int_{-\infty}^{\infty} x_1 Q(x_1) f_{x_1}(x_1) dx_1
\]


to account Equations (15) and (16), the scaling factor \( \kappa_1 \) is calculated by

\[
\kappa_1 = \frac{1}{\sigma_{x_1}^2} \int_{-\infty}^{\infty} x Q(x) f_{x_1}(x) dx \quad (17)
\]

Because no closed-form expression for the integral in Equation (17) is available, it is solved numerically. The variance of the output signal \( y_1[k] \) is given by

\[
\sigma_{y_1}^2 = E[y_1^2[k]] = \int_{-\infty}^{\infty} Q(x_1)^2 f_{x_1}(x_1) dx_1 \quad (18)
\]

Because \( x_1[k] \) and \( d_1[k] \) are uncorrelated with each other, the distortion noise variance is given by

\[
\sigma_{d_1}^2 = \sigma_{y_1}^2 - \kappa_1^2 \sigma_{x_1}^2 \quad (19)
\]

Similar to the presented results, the scaling factor \( \kappa_Q \) and the distortion noise variance \( \sigma_{d_0}^2 \) in the Q-branch are calculated.

Equation (17) shows that the scaling factors \( \kappa_1 \) and \( \kappa_Q \) can differ in case of unequal PDFs of \( x_1[k] \) and \( x_Q[k] \). Thus, a significant I/Q mismatch of \( \kappa_1 \) and \( \kappa_Q \) can only appear in the scenario 2. However, on the basis of the presented approach, a maximum I/Q mismatch of \( \kappa_Q / \kappa_1 \leq 0.35 \text{ dB} \) could be observed. This causes only a relatively small I/Q mismatch of the TXL interference and can be neglected. The impact of the \( \kappa_Q / \kappa_1 \) mismatch on the DL signal can be corrected digitally [14] and will be excluded in the following.

Ideally, the digital TXL compensation can cancel out perfectly the digitised TXL interference but not the increased distortion noise because it is uncorrelated to the TXL interference. Thus, in case of a perfect digital TXL compensation, the DL signal demodulation is distorted by the thermal noise and the distortion noise. Therefore, the SNR \( \gamma \) within the DL signal bandwidth after the analog-to-digital conversion and the correction of the VGA I/Q mismatch is given by

\[
\gamma = \frac{\left( \kappa_1^2 + \kappa_Q^2 \right) \sigma_x^2 \sigma_d^2}{\left( \kappa_1^2 + \kappa_Q^2 \right) \sigma_x^2 \sigma_d^2 + 2 \frac{\sigma_d^2 \sigma_{d_1}^2 + \frac{\sigma_d^2}{2}}{\left( \kappa_1^2 + \kappa_Q^2 \right) \sigma_x^2 \sigma_d^2}} \quad (20)
\]
Figure 4. Digital signal-to-noise ratio $\gamma$ without transmitter leakage and thermal noise ($\gamma = \infty$, $\gamma_{\text{TxL}} = \infty$, $\eta = 2$).

With the utilization of the definitions of $\tilde{y}$ in Equation (5) and $\gamma_{\text{TxL}}$ in Equation (4), Equation (20) can be rewritten as

$$
\gamma = \left[ \frac{1}{\tilde{y}} + \left( 1 + \frac{1}{g_w \tilde{y}} \right) \frac{2(x_1^2 \sigma_{d_1}^2 + x_2^2 \sigma_{d_2}^2)}{\eta (x_1^2 + \sigma_{Q}^2) \sigma_x^2} + \frac{4(x_1^2 \sigma_{d_1}^2 + x_2^2 \sigma_{d_2}^2)}{(1 + c_2^2)(x_1^2 + \sigma_{Q}^2) \eta g_w y_{\text{TxL}} \sigma_x^2} \right]^{-1}
$$

(21)

In the following, the analysis considers on a uniform mid-rise quantisation characteristic and an oversampling factor $\eta = 2$. In Figure 4, the SNR $\gamma$ in the digital frontend is plotted over the clipping ratio $\rho$ for the special case that no thermal noise and TxL interference is present at the VGA input ($\gamma = \infty$, $\gamma_{\text{TxL}} = \infty$). As can be seen, the digital SNR $\gamma$ is maximised by a bit resolution specific clipping ratio, which defines the variance $\sigma_x^2$ of the quantiser input signal. Applying these resolution-specific clipping ratios, Figure 5 shows the impact of the thermal noise and the TxL interference on the SNR $\gamma$. As can be seen, $\gamma$ flattens out below the analog SNR $\gamma$ at high $\gamma_{\text{TxL}}$. This SNR degradation is caused by the distortion noise, introduced by the quantisation. With decreasing $\gamma_{\text{TxL}}$, the VGA amplification drops; thus, the DL signal power at the ADC output decreases, whereas the distortion noise power remains constant and causes a degradation of $\gamma$. Furthermore, it can be seen that TxL deteriorates the digital SNR $\gamma$ much stronger in the transmission scenario 2 than in the scenario 1. This is caused by the approximately exponential distribution of the TxL interference. In low $\gamma_{\text{TxL}}$ ranges, the TxL interference dominates the PDF of the quantiser input signal and increases its kurtosis. Consequently, the PDF of $x_1[k]$ and $x_2[k]$ contain more probability mass around its mean and also in its tail than a Gaussian distribution with equal variance. This drives the quantiser more often in saturation and thus increases the variance of the distortion noise compared with the transmission scenario 1. Furthermore, Figure 5 shows that the analytical results match very accurately the numerical results, which validates the presented SNR analysis.

For the impact of TxL on the analog-to-digital conversion to be determined, Figure 6 depicts the SNR loss $\Delta \gamma$ of the digital SNR $\gamma$ with respect to a TxL-free transmission over the TxL SIR $\gamma_{\text{TxL}}$ for the two scenarios. As can be seen, the SNR loss can be reduced in low $\gamma_{\text{TxL}}$.

Figure 5. Impact of transmitter leakage on the signal-to-noise ratio $\gamma$ in the digital frontend ($\eta = 2$, $c_r = 0 \text{ dB}$).

DOI: 10.1002/ett
ranges only marginally by increasing the ADC bit resolution. Whereas an increase of $B$ by one bit reduces $\Delta y$ by approximately 3 dB in scenario 1, the SNR loss is lowered by only 1 dB in scenario 2. However, because increasing the bit resolution is strictly related with an increase of the power consumption and required chip area of the digital baseband processing, it can be assumed that the ADC bit resolution will not be increased in favour of a digital TxL compensation. Thus, the SNR loss limits the applicability of the digital TxL compensation. In case a SNR loss of 1 dB is tolerable for the DL transmission and $B = 8$ and $\gamma = 20$ dB is considered, the allowable TxL SIR is lower bounded by $\gamma_{\text{TxL}} = -16.7$ dB in scenario 1 and by $\gamma_{\text{TxL}} = 0$ dB in scenario 2. Lower TxL SIR levels require additional analog TxL mitigation approaches besides the digital TxL compensation.

Figures 4–6 assumed a perfect TxL I/Q match ($c_q = 1$). However, it can be shown by evaluating Equation (19) that the impact of $c_q$ on the digital SNR $\gamma$ is less than 0.5 dB.

4. TRANSMITTER LEAKAGE ESTIMATION

Because TxL is an additive interference, the digital TxL compensation is composed of the reconstruction and a subsequent subtraction of the TxL interference from the received signal $r[k]$. The reconstruction requires the estimation of the following parameters based on the knowledge of $s_{UL}[k]$ and $r[k]$; the equivalent, discrete CSF impulse responses $h_I[k]$ and $h_Q[k]$, the TxL I/Q mismatch $c_q$, the sign variable $c$, the equivalent, discrete TxL channel impulse response $h_{\text{TxL}}[k]$ and the DC offset compensation term $\epsilon$.

Temperature shifts and ageing of the circuitry affect the CSF transfer function in the I-branch and Q-branch. However, these fluctuations can be kept less than 1% using automatic tuning circuits [17]. Thus, the equivalent, discrete CSF impulse responses $h_I[k]$ and $h_Q[k]$ are only slowly time variant, which motivates their determination prior the TxL estimation using analog test signals while the Rx chain is in idle mode. The estimates of $h_I[k]$ and $h_Q[k]$ can be expressed as

$$
\hat{h}_I[k] = h_I[k] + w_{\hat{h}_I}[k], \quad \hat{h}_Q[k] = h_Q[k] + w_{\hat{h}_Q}[k] \tag{22}
$$

where $w_{\hat{h}_I}[k]$ and $w_{\hat{h}_Q}[k]$ shall be modelled as independent, real-valued Gaussian random variables having equal variances $\sigma^2$. The CSF filter length $L$ shall be assumed to be known. The TxL I/Q mismatch $c_q$ and the sign variable $c$ are properties of the considered transceiver circuitry and are only slowly time variant because of temperature shifts and ageing effects. Thus, the TxL I/Q mismatch can be determined using analog test signals, and its estimate can be written as

$$
\hat{c}_q = c_q + w_{\hat{c}_q} \tag{23}
$$

where $w_{\hat{c}_q}$ shall be modelled as a real-valued Gaussian random variable with the variance $\sigma_{\hat{c}_q}^2$. The estimation of the sign variable $c$ is very robust against the noise of the transceiver and is assumed to be known perfectly.

Moving objects in the near field of the transceiver antenna cause the time variance of the TxL channel [10]. Sections 4.1 and 4.2 present two approaches for the estimation of the TxL channel. The determination of the TxL

![Figure 6. Signal-to-noise ratio degradation in comparison with a transmission free from transmitter leakage (\(\eta = 2, c_q = 0\) dB).](image-url)
channel length $L_{\text{TXL}}$ is still an open research topic. One exemplary approach could be to estimate the TXL channel on the basis of a channel length hypothesis $\hat{L}_{\text{TXL}}$. In case of a too long chosen hypothesis, the tail of the estimated impulse response would be negligible small, motivating the decrement of the hypothesis. In the following, the perfect knowledge of $L_{\text{TXL}}$ is assumed.

On the basis of $\hat{h}_1[k]$, $\hat{h}_Q[k]$, $\hat{c}_q$ and $c$, the discrete baseband signal $r[k]$ can be preprocessed to simplify the TXL channel estimation. Because the CSF narrows the TXL interference, it is equalised in the first step. For the stability of the CSF equalisation to be ensured, linear, finite length equalisers are used, whose impulse responses $h_{\text{EQ},I}[k]$ and $h_{\text{EQ},Q}[k]$ of the length $L_{\text{EQ}}$ are adapted to $\hat{h}_1[k]$ and $\hat{h}_Q[k]$ according to the minimum mean square error (MMSE) criterion [18]. The combined impulse responses of the CSF and equaliser can be approximated by

$$
(\hat{h}_1*h_{\text{EQ},I})[k] \approx \delta[k-k_{\text{CSF}}]
$$

$$
(\hat{h}_Q*h_{\text{EQ},Q})[k] \approx \delta[k-k_{\text{CSF}}]
$$

(24)

where $\delta[k]$ denotes the Kronecker delta function. The equivalence in Equation (24) occurs in case of a perfect equalisation. It is worth to note that the CSF and subsequent equalisation introduces a time shift $k_{\text{CSF}}$ to the signals. Finally, the real and imaginary parts of the equalised, baseband signal are combined in the maximum ratio combining sense [18] according to the estimated TXL I/Q mismatch $\hat{c}_q$, yielding the real-valued received baseband signal.

$$
\hat{r}[k] = \frac{c}{1+\hat{c}_q} \left( (h_{\text{EQ},I}*r_I)[k] + \hat{c}_q (h_{\text{EQ},Q}*r_Q)[k] \right)
$$

$$
= \hat{b}[k] + \hat{w}[k]
$$

(25)

In case of a perfect CSF equalisation and a correct estimate of $\hat{c}_q$, the resulting TXL interference can be expressed by

$$
\hat{b}[k] = |h_{\text{TXL}}*s_{\text{UL}}|^2[k-k_{\text{CSF}}] + \hat{\epsilon}
$$

(26)

where $\hat{\epsilon}$ describes the DC offset compensation constant after the preprocessing. Consequently, Equation (25) rewrites as

$$
\hat{r}[k] = |h_{\text{TXL}}*s_{\text{UL}}|^2[k-k_{\text{CSF}}] + \hat{\epsilon} + \hat{w}[k]
$$

(27)

The received DL signal $(h_{\text{TXL}}*s_{\text{TX}})[k]$ and the noise signal $\hat{w}[k]$ are independent from the TXL interference and thus are combined in Equation (27) to the correlated noise signal $\hat{w}[k]$.

The TXL channel can be considered as quasi time invariant within a fraction of its coherence time. Therefore, the TXL estimation and compensation operate blockwise, where $N_T$ and $N_C$ denote the estimation and compensation block length, respectively. The TXL estimation delivers the estimates $\hat{h}_{\text{TXL}}[k]$ and $\hat{\epsilon}$, which are used to form the compensation signal

$$
s_c[k] = c (\hat{h}_1[k] + j \hat{c}_q \hat{h}_Q[k]) \ast (|\hat{h}_{\text{TXL}}*s_{\text{UL}}|^2[k] + \hat{\epsilon})
$$

(28)

and to subtract $s_c[k]$ from $r[k]$. For performance evaluation, let $\gamma_{\text{Est}}$ describe the SNR of the TXL estimation

$$
\gamma_{\text{Est}} = \frac{E{|\hat{b}[k]|^2}}{E{|\hat{w}[k]|^2}}
$$

(29)

describing the power ratio between the TXL interference $\hat{b}[k]$ and the combined noise signal $\hat{w}[k]$ after the described preprocessing.

The derivation of the TXL channel estimation approaches, presented in the next two subsections, assumes the perfect knowledge of $h_1[k]$, $h_Q[k]$ and $c_q$. Afterwards, this limitation will be dropped, and the performance of the estimation approaches is analysed for the case of an erroneous knowledge of these parameters.

### 4.1. Transmitter leakage estimation based on least mean square

Estimating the TXL channel impulse response, on the basis of the knowledge of $s_{\text{UL}}[k]$ and $r[k]$, is a nonlinear estimation problem due to the magnitude square operation in Equation (27). In case of a weak frequency selective TXL channel, its impulse response is dominated by one tap, that is $h_{\text{TXL}}[k] \approx c_{\text{TXL}} \delta[k-k_{\text{TXL}}]$. For its solution to be simplified significantly, the estimation problem in Equation (27) can be linearised by the following approximation:

$$
\hat{r}[k] \approx h_{\text{lin}}[k] \ast |s_{\text{UL}}[k-k_{\text{CSF}}]|^2 + \hat{\epsilon} + \hat{w}[k]
$$

(30)

where $h_{\text{lin}}[k] = |c_{\text{TXL}}|^2 \delta[k-k_{\text{TXL}}]$. Thus, only the delay $k_{\text{TXL}}$ and the magnitude square of the dominating TXL channel tap $|c_{\text{TXL}}|^2$ have to be estimated. For convenience, the TXL interference in Equation (30) shall expressed as vector product

$$
\hat{r}[k] = h_{\text{lin}}^T \cdot s_{\text{UL,IM2}}[k-k_{\text{CSF}}] + \hat{\epsilon} + \hat{w}[k]
$$

(31)

with $h_{\text{lin}} = [h_{\text{lin}}[0], \ldots, h_{\text{lin}}[L_{\text{TXL}}-1]]$, $\epsilon$ and $s_{\text{UL,IM2}}[k] = [|s_{\text{UL}}[k]|^2, \ldots, |s_{\text{UL}}[k-L_{\text{TXL}}+1]|^2, 1]^T$.

The LMS filter approach [18] is designed for solving linear estimation problems. Therefore, the authors of this work proposed in [8] the use of the LMS approach for solving the linearised TXL estimation problem formulated in Equation (31). The LMS estimates recursively the parameter vector $h_{\text{lin}}[k]$.

$$
h_{\text{lin}}[k] = \hat{h}_{\text{lin}}[k-1] + 2 \mu E \{ e[k-1] s_{\text{UL,IM2}}[k-k_{\text{CSF}}] \}
$$

(32)

where $e[k-1]$ denotes the estimation error based on the previous estimate:

$$
e[k-1] = \hat{r}[k] - \hat{h}_{\text{lin}}^T[k-1] \cdot s_{\text{UL,IM2}}[k-k_{\text{CSF}}]
$$

(33)
The expectation in Equation (32) is usually not known and is approximated by averaging over the several samples. However, in practice, only one sample is used to approximate the expectation to achieve a very low complexity. Therefore, the estimator uses a stochastic gradient, reasoning its name *stochastic gradient* LMS estimator. The recursive estimation in Equation (32) simplifies to

$$\hat{h}_{\text{LMS}}[k] = \hat{h}_{\text{lin}}[k-1] + 2\mu \, e[k-1] \, s_{\text{UL,IM2}}[k-k_{\text{CSF}}]$$  (34)

Thus, $N_{E}$ parameter vectors are estimated in each estimation block. Because of the stochastic properties of the gradient and the approximation errors in the linear formulation of the estimation problem in Equation (30), the estimated parameter vectors $\hat{h}_{\text{lin}}[k]$ walk randomly around the MMSE estimate. Thus, averaging over the $N_{E}$-estimated parameter vectors is a suitable approach getting closer to the MMSE. If the current estimation block starts at $k = 0$, the averaging yields

$$\hat{h}_{\text{LMS}} = \frac{1}{N_{E}} \sum_{k=0}^{N_{E}-1} \hat{h}_{\text{lin}}[k]$$  (35)

According to its definition, the vector $\hat{h}_{\text{LMS}}$ contains the $L_{\text{Txl}}$ taps of the estimated channel impulse response $h_{\text{lin}}[k]$ and the DC offset compensation constant $\hat{\varepsilon}$. Because of estimation and approximation errors, $h_{\text{lin}}[k]$ contains more than one nonzero tap in the case of a frequency selective as well as frequency flat TxL channel. However, the linear approximation in Equation (31) considers only the strongest tap for the TxL compensation, whose delay can be estimated by

$$\hat{k}_{\text{Txl}} = \arg \max_{m} \left| \hat{h}_{\text{LMS}}[m] \right|, \quad m = 0, \ldots, L_{\text{Txl}} - 1$$  (36)

Although only the value and the delay of this dominating tap need to be determined, the LMS assumes a frequency-selective impulse response $h_{\text{lin}}[k]$ for estimating the delay $\hat{k}_{\text{Txl}}$. For the estimation performance to be improved, the LMS estimator is used in a second iteration to determine solely the value of the dominating tap and the DC offset compensation constant. Therefore, the LMS operates in the second iteration on the system model in Equation (31), where the vectors are redefined to $\hat{h}_{\text{lin}} := \left[ |C_{\text{Txl}}|^2, \hat{\varepsilon} \right]^T$ and $s_{\text{UL,IM2}}[k] := \left[ s_{\text{UL}}[k - \hat{k}_{\text{Txl}}], 1 \right]^T$. The vector $\hat{h}_{\text{lin}}$ is estimated according to Equations (33) and (34), where the parameter vector is initialised with the previous estimates $\hat{h}_{\text{LMS}}$. Subsequently, the estimated $N_{E}$ parameter vectors $\hat{h}_{\text{lin}}[k]$ are averaged to minimise the impact of estimation errors, yielding

$$\hat{h}_{\text{LMS}} = \frac{1}{N_{E}} \sum_{k=0}^{N_{E}-1} \hat{h}_{\text{lin}}[k]$$  (37)

Thus, the estimates of the dominating TxL channel tap magnitude and the DC offset compensation constant are given by $\hat{c}_{\text{Txl}} = \sqrt{\hat{h}_{\text{LMS}}[0]}$ and $\hat{\varepsilon} = \hat{h}_{\text{LMS}}[1]$, respectively. The TxL interference is reconstructed by

$$x_{\text{C}}[k] = (\hat{h}_{1}[k] + j \hat{c}_{q} \hat{h}_{Q}) \ast \left( |\hat{c}_{\text{Txl,UL}}[k - \hat{k}_{\text{Txl}}]|^2 + \hat{\varepsilon} \right)$$  (38)

and subtracted as compensation signal from the received signal $r[k]$.

### 4.2. Transmitter leakage estimation based on least squares

Because the LMS estimator approach estimates only the mean TxL channel attenuation, its estimation accuracy deteriorates severely in case of frequency selective TxL channels. Therefore, this subsection presents a second TxL channel estimation approach suitable for frequency flat and frequency selective TxL channels. It was first proposed in [19] and shall be reviewed here.

As identified in the previous subsection, determining the TxL channel is a nonlinear estimation problem. Unfortunately, this type of estimation problems is much more complicated to solve than its linear counterpart. Therefore, the estimation problem (27) is reformulated to a linear problem at the expense of a larger parameter vector to be estimated. On the basis of the knowledge of the TxL channel length $L_{\text{Txl}}$, the TxL interference $\tilde{b}[k]$ in Equation (27) can be rewritten as

$$\tilde{b}[k] = \hat{\varepsilon} + \sum_{m_1, m_2 = 0}^{L_{\text{Txl}} - 1} h_{\text{Txl}}[m_1] \, h_{\text{Txl}}^*[m_2] \, s_{\text{UL}}[k - k_{\text{CSF}} - m_2]$$  (39)

where $\sum_{m_1, m_2}$ denotes a double sum with equal limits. With the utilization of the equivalences $\Re \{ z \} = (1/2)(z + z^*)$ and $\Im \{ z \} = (1/2j)(z - z^*)$, $z \in \mathbb{C}$, the summends can be expressed as element-wise multiplication of two matrices $S[k]$ and $G \in \mathbb{R}^{(L_{\text{Txl}} \times L_{\text{Txl}})}$:

$$\tilde{b}[k] = \hat{\varepsilon} + \sum_{m_1, m_2 = 0}^{L_{\text{Txl}} - 1} S[m_1, m_2] \, [k - k_{\text{CSF}}] \, G[m_1, m_2]$$  (40)

where the elements of $S[k]$ are defined by the UL signal $s_{\text{UL}}[k]$

$$S[m_1, m_2] = \begin{cases} \Re \{ s_{\text{UL}}[k - m_1] \, s_{\text{UL}}^*[k - m_2] \} & m_1 \leq m_2 \\ \Im \{ s_{\text{UL}}[k - m_1] \, s_{\text{UL}}^*[k - m_2] \} & m_1 > m_2 \\ \end{cases}$$  (41)

and the elements of $G$ are derived from $h_{\text{Txl}}[k]$

$$G[m_1, m_2] = \begin{cases} 2 \Re \{ h_{\text{Txl}}[m_1] \, h_{\text{Txl}}^*[m_2] \} & m_1 \leq m_2 \\ -2 \Im \{ h_{\text{Txl}}[m_1] \, h_{\text{Txl}}^*[m_2] \} & m_1 > m_2 \\ \end{cases}$$  (42)

By rearranging $S[k]$ and $G$ as column vectors $s[k]$ and $g \in \mathbb{R}^{(L_{\text{Txl}}^2 \times 1)}$, respectively, Equation (40) can be rewritten as

$$\tilde{b}[k] = s[k - k_{\text{CSF}}] \, g + \hat{\varepsilon}.$$  (43)
The DC offset compensation constant $\bar{\varepsilon}$ can easily be integrated in the estimation problem by extending $\mathbf{g}$ and $s[k]$ accordingly, yielding the vectors

$$\tilde{s} = \begin{bmatrix} \mathbf{s} \\ 1 \end{bmatrix}, \quad \tilde{\mathbf{g}} = \begin{bmatrix} \mathbf{g} \\ \bar{\varepsilon} \end{bmatrix} \quad (44)$$

Hence, the TxL interference can be expressed as a scalar product of a known signal vector $\tilde{s}[k]$ and an unknown coefficient vector $\tilde{\mathbf{g}}$, and Equation (27) rewrites as

$$\tilde{r}[k] = \tilde{s}[k-k_{\text{CSF}}]^T \tilde{\mathbf{g}} + \tilde{w}[k] \quad (45)$$

Evidently, some elements of $\tilde{\mathbf{g}}$ are correlated with each other. Exploiting this correlation would transform the estimation problem in a nonlinear one. However, because we aim at a linear problem formulation to simplify its solution, the dependencies in $\tilde{\mathbf{g}}$ are not taken into account by the estimator.

As mentioned before, the TxL estimation operates block-wise. Therefore, the $N_{\text{E}}$ samples of $\tilde{r}[k]$ and $\tilde{w}[k]$ of the current block form the vectors $\mathbf{r}[k] = [\tilde{r}[k], \ldots, \tilde{r}[k - N_{\text{E}} + 1]]^T$ and $\mathbf{w}[k] = [\tilde{w}[k], \ldots, \tilde{w}[k - N_{\text{E}} + 1]]^T$ and the $N_{\text{E}}$ signal vectors form the signal matrix $\tilde{\mathbf{S}}[k-k_{\text{CSF}}] = [\tilde{s}[k-k_{\text{CSF}}], \ldots, \tilde{s}[k-k_{\text{CSF}}] - N_{\text{E}} + 1]]^T$. Thus, the vector $\tilde{r}[k]$ can be expressed as the vector–matrix product

$$\tilde{r}[k] = \tilde{\mathbf{S}}[k-k_{\text{CSF}}]^T \tilde{\mathbf{g}} + \tilde{\mathbf{w}}[k] \quad (46)$$

Because one major advantage of the digital TxL compensation is the independence from the currently supported communication standard, no assumptions on the stochastic properties of the UL signal can be made. Therefore, the coefficient vector $\tilde{\mathbf{g}}$ is estimated using the LS approach. This estimator is designed for linear estimation problems. It estimates the parameter set, which minimizes the squared error between the observed signal and the reconstructed signal. Thus, the LS-based TxL estimator determines the coefficient vector $\tilde{\mathbf{g}}$, which minimizes the Euclidean distance between $\tilde{r}[k]$ and the reconstructed TxL interference. Recalling that the correlations between the elements of the coefficient vector are neglected, the LS estimate of $\tilde{\mathbf{g}}$ is given by [20]

$$\tilde{\mathbf{g}} = (\tilde{\mathbf{S}}[k-k_{\text{CSF}}]\tilde{\mathbf{S}}[k-k_{\text{CSF}}]^T)^{-1} \tilde{\mathbf{S}}[k-k_{\text{CSF}}] \tilde{\mathbf{r}}[k] \quad (47)$$

The invertibility of $\tilde{\mathbf{S}}[k-k_{\text{CSF}}]\tilde{\mathbf{S}}[k-k_{\text{CSF}}]^T$ is guaranteed by the following reasoning. Section 5 shows that the estimation block length must be chosen much larger than the size of the coefficient vector ($N_{\text{E}} \gg L_{\text{TTL}}^2 + 1$) to achieve reliable estimation results. Because of the stochastic of $\mathbf{s}_{\text{TTL}}[k]$ and the large block length, $\tilde{\mathbf{S}}[k]$ can be assumed to have full rank $L_{\text{TTL}}^2 + 1$ and thus being positive semi definite. This property ensures the invertibility of the matrix. With the utilization of $\tilde{\mathbf{g}}$, the TxL interference $\tilde{b}[k]$ is estimated by

$$\tilde{b}[k] = \tilde{s}[k - \Delta k]^T \tilde{\mathbf{g}} \quad (48)$$

and the corresponding compensation signal is given by

$$s_c[k] = c(\hat{h}_1[k] + j\hat{c}_q[h_Q[k] \tilde{b}[k]] \quad (49)$$

As will be shown in the next section, it might be beneficial to neglect the last taps of $h_{\text{TTL}}[k]$ in the TxL estimation and by this to reduce the size of the coefficient vector $\mathbf{g}$. Thus, let $L_{\text{TxL}}$ denote the TxL channel length considered by the TxL estimator. Consequently, the signal matrix $\tilde{\mathbf{S}}[k]$ in Equation (47) is constructed according to $L_{\text{TxL}}$.

5. NUMERICAL RESULTS

For performance evaluation of the LMS and LS estimation approaches, a FDD simulation chain is used, modelling a 2.5 MHz LTE similar to OFDM transmission with 126 data subcarriers and 22 pilot subcarriers in the DL and UL. The data subcarriers are modulated by 64-QAM and 16-QAM symbols in the DL and UL, respectively. For channel coding, a rate 1/2 convolutional code with generator polynomial $G = [133, 171]_8$ is used with a code word length of one OFDM symbol. The DL channel and the TxL channel are assumed to exhibit block fading and are modelled according to the International Telecommunication Union Vehicular A model and the TxL channel model, given by Equation (2) with $\tau = 1$ and $\tau = 4$, respectively. For channel select filtering, two identical fourth-order Chebychev filters are used, approximated by eight-tap finite impulse response filters. The ADC employs an $\eta = 2$ oversampling with a sampling frequency $f_s = 7.68$ MHz and an uniform 8-bit mid-rise quantisation. The TxL estimation applies the preprocessing, described in Equation (25), and uses two 65-tap CSF equalisation filters. The performance results, presented in the following, assume a perfect knowledge of $h_1[k]$, $h_Q[k]$ and $c_q$ in the TxL estimation and a perfect knowledge of the DL channel with a zero forcing equalisation in the Rx chain. In Figure 7(a) and (b), the compensation gain of the LMS and LS estimator is plotted over the SNR of the TxL estimation $\gamma_{\text{Est}}$ and the estimation block length $N_{\text{E}}$, respectively. As Figure 7(a) shows, in case of a weak frequency selective TxL channel ($\tau = 4$), the LMS achieves very accurate estimation results and can reduce the TxL interference for $N_{\text{E}} = 4000$ by up to 19.8 dB. The approximation errors, contained in the LMS approach, and the imperfect CSF equalisation inhibit a higher estimation precision, and cause the compensation gain, to flatten out in high $\gamma_{\text{Est}}$ regions. With increasing frequency selectivity of the TxL channel, the approximation errors deteriorates severely the LMS performance. The compensation gain of the LS approach increases first linearly with $\gamma_{\text{Est}}$ and flattens out at high $\gamma_{\text{Est}}$ as a consequence of the imperfect CSF equalisation. Furthermore, the figure shows that the LS is very robust against frequency selectivity of the TxL channel. Because in practice the TxL channel frequency selectivity is not known a priori, this robustness is an outstanding advantage of the LS compared with the LMS.

In Figure 7(b), the compensation gain is plotted over the estimation block length for the case of a frequency

DOI: 10.1002/ett
selective TxL channel ($\tau = 1$). As can be seen, after a linear rise the compensation gain of the LS flattens out as consequence of the CSF equalisation errors. The LMS levels out much earlier because of the approximation errors embodied in the estimation approach.

In Figure 8, the coded frame error rate (FER) of the DL transmission is plotted for a TxL SIR $\gamma_{\text{TxL}} = 10$ dB. As mentioned in Section 1, the TxL interference causes an additional, irreversible SNR loss to the DL signal. For this effect in the curves to be identified, the definition of $E_b/N_0$ does only take into account the thermal noise and not the ADC quantisation noise. Figure 8 shows that TxL degrades severely the FER performance. The LMS achieves significant improvement depending on the TxL channel frequency selectivity. However, the LS achieves a huge performance improvement for both TxL channels.

For the characterisation of the sensitivity of the OFDM transmission to TxL and the performance improvements achieved by the presented compensation approaches, Figure 9(a) and (b) show the SNR loss with respect to a TxL-free transmission at a target FER 1% as a function of $\gamma_{\text{TxL}}$ for the case of a strong and weak frequency selective TxL channel ($\tau = 1$ and $\tau = 4$). Let consider first the case of a frequency selective TxL channel. As Figure 9(a) shows, the OFDM transmission is susceptible to TxL for $\gamma_{\text{TxL}} < 30$ dB when no TxL compensation is employed. The additional SNR loss, caused by the quantisation of TxL, limits the applicability of the digital TxL compensation. The black solid curve at the left of Figure 9(a) represents the case of a complete compensation of the digitised TxL interference (Genie Comp.). Thus, the solid and dashed black curves represent the upper and lower bound of the digital TxL compensation operating range. Let us consider first the case that the total TxL channel length is considered in the TxL estimation ($L_{\text{TxL}} = L_{\text{TxL}}$). Whereas the LMS performance degrades severely already for relatively high values of $\gamma_{\text{TxL}}$ and causes large SNR losses, the LS still achieves accurate estimation results and can effectively limit the SNR loss. If an SNR loss of $1.2$ dB at a FER of 1% is acceptable in the DL transmission, the employment of the LMS approach allows an increase of the TxL interference in the analog frontend by up to 5 dB compared with the case without any TxL compensation. In contrast to this, the LS allows an increase of the TxL interference by
up to 22.7 dB and becomes as close as 6.1 dB to the case of a perfect TxL compensation. This gap can be reduced by an increase of the estimation block length \( N_E \). Additionally, Figure 9(a) shows the SNR loss of the LS approach, when only the first \( L_{TXL} < L_{TXL} \) taps of the TxL channel are estimated and compensated. Thus, the amount of coefficients to be estimated is reduced, yielding a more precise estimation and reducing the minimal SNR loss. However, with decreasing \( \gamma_{TXL} \), the SNR loss increases faster than for the case \( L_{TXL} = L_{TXL} \), according to the energy of the neglected TxL channel taps.

Figure 9(b) shows the SNR loss for the case of a weak frequency selective TxL channel (\( \tau = 4 \)). The employment of the LMS relaxes the TxL requirements on the analog frontend by 17.9 dB and for the LS by 22.7 dB. Neglecting the last two channel taps, the LS relaxes the TxL requirements by 25.1 dB. It should be noted that a relaxation of the TxL requirements by \( x dB \) can be used for a relaxation of the receiver IIP2 by \( x dB \) or a relaxation of the required Tx–Rx isolation of the duplexer by \( x/2 dB \).

Finally, Figure 10(a) shows the compensation gain of the presented TxL estimation and compensation approaches for different TxL channel length and an intensional neglect of the last channel taps in the estimation (\( L_{TXL} < L_{TXL} \)).

As the figure shows, the compensation gain of the LS slightly increases with decreasing channel length \( L_{TXL} \). Because the LMS considers inherently only one tap, its performance does not change with \( L_{TXL} \). For the case of \( L_{TXL} < L_{TXL} \), the LS achieves an improvement of \( G_C \) in the low \( \gamma_{TXL} \) regime. At high TxL SIR \( \gamma_{TXL} \), the energy of the neglected channel taps reduces the maximal achievable compensation gain. However, because the TxL estimation must operate in a very noisy environment [2], neglecting the last TxL channel taps is beneficial in two ways: the computational complexity is reduced and the compensation gain increases.

So far, the perfect knowledge of the TxL I/Q mismatch \( c_q \) and the equivalent, discrete CSF impulse response \( h_I[k] \) and \( h_Q[k] \) were assumed. In practice, only estimates of these parameters are available (e.g., by calibration using test tones). Therefore, the sensitivity of the presented TxL estimation approaches to these estimation errors is analysed in the following. For the comparability of the results to be ensured, the relative variance \( \delta_h^2 \) and \( \delta_{c_q}^2 \) of the CSF estimation error in Equation (22) and the TxL I/Q mismatch estimation error in Equation (23) are considered, respectively

\[
\delta_h^2 = \frac{\sigma_h^2}{L \sum_{m=0}^{L-1} |h_I[m]|^2} = \frac{\sigma_h^2}{L \sum_{m=0}^{L-1} |h_Q[m]|^2} \quad (50)
\]

\[
\delta_{c_q}^2 = \frac{\sigma_{c_q}^2}{c_q^2}
\]

Figure 10(b) shows the degradation of \( G_C \) with \( \delta_h^2 \) and \( \delta_{c_q}^2 \) for the case that the TxL I/Q mismatch estimation error and the CSF estimation error is present solely, respectively.

As can be seen, the LMS and the LS are very robust against such estimation errors, and their \( G_C \) performance degrades only slightly at high estimation error variances.

For the evaluation of the presented digital TxL compensation approaches to be completed, their computational complexity shall be compared coarsely in terms of number.
Table I. Comparison of transmitter leakage estimation approaches based on least mean squares and least squares.

<table>
<thead>
<tr>
<th></th>
<th>LMS</th>
<th>LS</th>
</tr>
</thead>
<tbody>
<tr>
<td>TxL compensation gain</td>
<td>5–17.9 dB</td>
<td>22.7 dB</td>
</tr>
<tr>
<td>(TxL channel (\tau = 1−4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sensitivity to frequency selectivity of TxL channel</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Sensitivity to block length (N_E)</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Complexity</td>
<td>Low</td>
<td>High</td>
</tr>
</tbody>
</table>

TxL, transmitter leakage; LMS, least mean squares; LS, least squares.

Figure 10. Compensation gain \(G_c\) of the proposed transmitter leakage compensation approaches \((\tau = 1, N_E = 4000)\).

6. CONCLUSIONS

This work presented a system model, describing the impact of TxL on the digital received signal in zero-IF receivers. The TxL impact on the analog-to-digital conversion in the receiver is analysed, and it was shown analytically that TxL causes an additional irreversible SNR loss of the desired, received signal. This SNR degradation limits the applicability of a digital TxL compensation. However, the SNR loss remains negligibly small as long as the TxL interference does not exceed significantly the desired received signal.

The digital TxL compensation operates directly on the ADC output signal by using the discrete input signal of the DAC in the Tx chain. Furthermore, it does not draw any assumptions on the UL and DL signals. Thus, the digital TxL compensation is independent of the supported communication standard, and therefore, it is highly attractive for modern multiband, multistandard transceivers. The main challenge of the digital TxL compensation can be traced back to the estimation of the TxL channel. For this purpose, two estimation approaches were presented. The first bases on a LMS filter and has a very low computational complexity. It approximates the TxL channel as frequency cycles for each time sample \(r[k]\), whereas the LS-based estimator requires 107 967 computation cycles [4]. A summary of the properties of the LMS-based and the LS-based TxL estimation approaches is given in Table I.
flat and thus is suitable for frequency flat and weak frequency selective channels. Assuming a uniform 8-bit mid-rise ADC and an estimation block length of $N_E = 4000$, the employment of the LS relaxes the requirements on the analog frontend with respect to TxL by 17.9 and 5 dB in case of a weak and strong frequency selective six-tap TxL channel, respectively.

The second approach uses an LS estimator and is suitable for frequency selective TxL channels in general. Independent from the TxL channel frequency selectivity, it achieves a higher estimation accuracy than the LMS at the expense of a higher computational complexity. Assuming a six-tap TxL channel, a uniform 8-bit mid-rise ADC and an estimation block length of $N_E = 4000$, the employment of the LS relaxes the requirements on the analog frontend with respect to TxL by 22.7 dB. In practice, the frequency selectivity of the TxL channel is not known a priori. Therefore, the LS approach is more attractive for the practical application than the LMS because it offers a higher reliability of the estimation results.

Using these TxL compensation approaches relaxes significantly the requirements on the analog frontend and, by this, simplifies the design of a frequency agile analog frontend, which are highly attractive for modern multistandard transceivers.

REFERENCES


AUTHORS’ BIOGRAPHIES

Andreas Frotzscher received his Dipl.-Ing. (MSEE) and PhD degree in Electrical Engineering from TU Dresden, Germany in 2005 and 2010, respectively. From 2005 to 2010, he was a research associate at the Vodafone Chair Mobile Communications Systems at TU Dresden, Germany. Since 2010, he is working at the Bell Labs of
Alcatel-Lucent in Stuttgart, Germany. His research interests focus on physical layer signal processing, transceiver design, nonlinear estimation techniques and radio-over-fibre transceiver concepts.

Gerhard Fettweis earned his PhD from RWTH Aachen in 1990. Thereafter, he was at IBM Research in San Jose, CA, USA and then at TCSI Inc., Berkeley, CA, USA. Since 1994, he is a Vodafone Chair Professor at TU Dresden, Germany, with currently 20 companies from Asia/Europe/US sponsoring his research on wireless transmission and chip design. He runs the world’s largest cellular research test bed in downtown Dresden (EASY-C). In Dresden, he has spun-out nine start-ups so far and setup funded projects of more than EUR 1/4 billion volume. He has been actively involved in organising IEEE conferences, most notably being the TPC Chair of IEEE ICC 2009 (Dresden).