Robust Precoding with General Power Constraints Considering Unbounded Channel Uncertainty

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Abstract—In this contribution we deal with a cooperative cellular downlink scenario, where collaborating base stations jointly serve multiple users in a multiple-input multiple-output fashion. Linear spatial signal processing filters are applied at transmitter and receiver. The filters are designed in order to optimize four different mean square error related objective functions, considering general power constraints, i.e., transmit power constraints per arbitrary group of antennas. This optimization is based on channel state information, which is only imperfectly known in practical setups. In this contribution, we present a filter design for the stated optimization problems, taking statistical knowledge of unbounded channel uncertainty into account.

I. INTRODUCTION

Linear joint transmission in cooperative cellular networks provides substantial performance gains, especially for users close to the cell edge [1]. Coordinated multi-antenna base stations (BSs) together with multi-antenna user equipments (UEs) constitute a distributed multiuser–multiple-input multiple-output (MU–MIMO) system capable of spatial multiplexing, i.e., transmitting multiple user data streams in parallel using the same transmission resource. In the downlink, interference towards multiple UEs is handled at the transmitter side (precoding), which makes the optimization more challenging since precoding vectors are coupled.

Linear spatial signal processing is a technique to realize MU-MIMO with reasonable complexity compared to better performing non-linear methods [2]. In this context mean square error (MSE) optimization is an established technique for linear spatial filter design due to its tractable mathematical structure and its connection to capacity [3]. First contributions focus on minimizing the sum user MSE under a sum power constraint and uniform power allocation to single antenna UEs [4]. Performance gains can be achieved by allocating non-uniform transmit powers among UEs [5]. An extension to multi-antenna UEs was presented in [6]. Other contributions employ objective functions like power minimization with sum MSE [7], targets or maximum user MSE related optimization [8]. In distributed setups transmit power is restricted per BS and possibly per individual antenna. These type of constraints is referred to as general power constraints [9]. MSE related optimization considering cooperative networks was presented in [10].

The basic requirement for realizing joint transmission is the availability of channel state information (CSI) at each of the collaborating BSs. Due to a variety of impacts (e.g., channel estimation or feedback quantization) CSI is only imperfectly known in practice, leading to substantial performance losses [11]. However, robustness can be introduced by incorporating statistical knowledge of the channel uncertainty into the spatial filter design [12], [13]. In this paper we focus on robust filter design with general power constraints considering the four optimization problems:

- Minimizing the sum MSE under general power constraints
- Minimizing the overall transmit power under a target sum MSE and general power constraints
- Minimizing the maximum weighted user MSE under general power constraints
- Minimizing the overall transmit power under per user target MSEs and general power constraints

These problems were already addressed for perfect CSI [10] and bounded channel uncertainty [14]. In this paper we extend this work by deriving solutions for unbounded channel uncertainty, which is more relevant for practical setups.

The remainder of this paper is structured as follows. In Section II we introduce the system model and state the mathematical problem formulations. Section III discusses the filter optimization while simulation results are presented in Section IV before the paper is concluded in Section V.

Throughout this paper we denote matrix transposition, conjugate transposition and expectation with $(\cdot)^T$, $(\cdot)^H$ and $E\{\cdot\}$, respectively. The matrix operator $\text{vec}(A)$ generates a vector by stacking the columns of $A$ one below the other, while $\text{blkdiag}\{A_1, \ldots, A_N\}$ places the matrix blocks $A_1, \ldots, A_N$ in the diagonal of a matrix. We further use $\otimes$, $||\cdot||_2$, $\text{tr}(\cdot)$ and $\text{dg}(\cdot)$ for Kronecker product, Frobenius norm, trace and diagonalization, respectively.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We focus on a system where $M$ cooperating BSs jointly transmit to $K$ UEs. Each BS $m \in M$ is equipped with $B_m$ transmit antennas and each UE $k \in K$ with $U_k$ receive antennas. The sets of BS and UE indices are defined as $M = \{1, \ldots, M\}$ and $K = \{1, \ldots, K\}$, respectively. The overall number of BS antennas results in $B = \sum_{m \in M} B_m$, while the overall number of UE antennas is $U = \sum_{k \in K} U_k$.

A. Transmission Model

We assume that the data symbol vector $d_k = [d_k^H, \ldots, d_K^H]^H$ including the data symbols $d_k \in \mathbb{C}^{U_k \times 1}$ of all jointly served users is perfectly known at every BS.
The number of data streams assigned to each UE $k$ is equal to its receive antennas $U_k$. On that account, $U \leq B$ is required. Without loss of generality we consider independent and identically distributed (i.i.d.) Gaussian data symbols with zero mean and unit variance $E[|d|^2] = I$. In order to reduce the interference observed at the UEs the data symbols are spatially pre-processed before transmission. We consider linear precoding, where $d$ is multiplied with the precoding matrix $B = [B_1^H, \ldots, B_M^H]^H$ and $B_m \in \mathbb{C}^{B_m \times U}$ is the partition of $B$ applied at each BS $m$. Additionally, $B_k \in \mathbb{C}^{B \times \ell_K}$ is the partition of $B = [B_1, \ldots, B_K]$ related to each UE $k$. The precoded symbols are transmitted to the UEs considering frequency flat channels. The coupling between the BS and the UEs is captured by the channel matrix $H = [H_1^H, \ldots, H_K^H]^H$, where $H_k = [H_{k,1}, \ldots, H_{k,m}] \in \mathbb{C}^{U_k \times B}$ is the channel from all $M$ BS antennas to each UE $k$. Assuming all spatial links between BS $m$ and UE $k$ are uncorrelated and provide the same mean channel gain $\gamma_{k,m}$ (related to path loss and shadow fading), the channel covariance matrix can be written as $E[H_{k,m}H_{k,m}^H] = \gamma_{k,m}B_mB_k^H$. The received signal vector at UE $k$ is impaired by additive white Gaussian noise (AWGN) $n_k \sim \mathcal{N}_C(0, \sigma_n^2I)$ before it is equalized with the receive filter $U_k$. The joint data symbols of all $K$ UEs after user-wise equalization results in

$$d = U(HBd + n),$$

where $U = \text{blkdiag}\{U_1, \ldots, U_K\}$ and $n = [n_1^H, \ldots, n_K^H]^H$. The transmit power at BS $m$ is restricted to $\text{tr}(BB_m^H) \leq \rho_m$, $\forall m \in M$. For calculating $B$ and $U$ the complete channel matrix $H$ need to be known either at all BSs or at a central unit connected to all BSs. Due to impairments like channel estimation errors or feedback quantization, the channel is only imperfectly known at the BSs and can be interpreted as a random variable

$$H = \hat{H} + E$$

with mean $\hat{H}$ (channel estimate) and covariance $\Phi_E = E\{|EE|^H\}$ (channel uncertainty).

### B. Feedback Model

In this subsection we introduce a model for $\hat{H}$ and $E$ considering channel estimation and quantized feedback transmission. Note, that derivations in Section III are not restricted to this model.

It is considered that pilots from all $M$ BSs are received at each UE $k$ and fed back to the BS the UE is assigned to. Afterwards CSI is forwarded to the other BSs or a central unit without further impairments, based on the idealistic assumption of latency free backhauling. Aspects regarding backhaul latency are discussed, e.g., in [15].

It is assumed that the pilot signals

$$\hat{y}_{k,m} = h_{k,m} + \tilde{n}_{k,m},$$

received at UE $k$ are corrupted by the AWGN $\tilde{n}_{k,m} \sim \mathcal{N}_C(0, \sigma_n^2/\rho_pI)$, where $\rho_p$ reflects the product of pilot power and number of pilots within a transmission block experiencing a static channel. Furthermore, we define $h_{k,m} = \text{vec}(H_{k,m}) \sim \mathcal{N}_C(0, \gamma_{k,m}I)$, $\forall k, m$. In order to improve readability we omit the indices $k$ and $m$ in the remainder of this section.

1) **Quantization**: The received signal $\hat{y}$ needs to be fed back to the base station with finite feedback rate, due to limited uplink resources. The effect of quantization of a complex Gaussian random variable with $Q$ bits can be modeled with rate distortion theory [16]. With this approach, the output of the quantizer results in the superposition of the input scaled by $a = 1 - 2^{-Q}$ and the AWGN $q \sim \mathcal{N}_C(0, \Phi_q)$ with variance

$$\Phi_q = 2^{-Q} \left(1 - 2^{-Q}\right) \left(\gamma + \frac{\sigma_n^2}{\rho_p}\right) I.$$ (4)

Considering uncorrelated spatial links with equal mean channel gains, uniform quantization (the same number of quantization bits for each link) is optimal.

2) **Estimation**: Based on the side information

$$y = a(h + \tilde{n}) + q$$

available at the BS, the channel is estimated by maximizing the conditional probability density function (PDF) $p_{\hat{h}|y} = \mathcal{N}_C(h, \Phi_{h|y})$ by choosing its mean

$$\hat{h} = E\{|Hy|^H\}E^{-1}\{yy\}y = \left(1 + \frac{\sigma_n^2}{\gamma_{\rho_p}}\right)^{-1}y,$$ (6)

corresponding to minimum MSE estimation [17]. The channel uncertainty results in

$$\Phi_{h|y} = E\{|hHy|^H\} - E\{|hHy|^H\}E\{|yy|^H\}^{-1} = \gamma \left(\frac{2-a+2\sigma_n^2/(\gamma_{\rho_p})}{1+\sigma_n^2/(\gamma_{\rho_p})}\right) I = \sigma_k^2I.$$ (7)

From this result the actual channel known at the BS can be interpreted as a Gaussian random vector with the PDF $p_{h|y} = \mathcal{N}_C(h, \sigma_k^2I) = \hat{h} + e$, which can also be displayed as the sum of the deterministic vector $\hat{h}$ and a zero mean random vector $e \sim \mathcal{N}_C(0, \sigma_k^2I)$. Transforming the vectorized notation back to the original matrix notation captures (2) with

$$\Phi_E = E\{|EE|^H\} = B\sigma_k^2I.$$ (8)

Reflecting different channel uncertainties for different links, the general notation can be written as

$$\Phi_E = \text{blkdiag}\{\psi_1I_{U_1}, \ldots, \psi_KI_{U_K}\}$$ (9)

where $\psi_k = \sum_{m=1}^MB_m\sigma_k^2[k,m]$ is the sum channel uncertainty of UE $k$. 

![Feedback chain including noisy pilot reception, quantization and estimation at the BS.](Fig. 1)
C. Problem Formulation

Next we discuss the four MSE related optimization problems addressed in I. The sum MSE can be written as

$$
\epsilon_{\text{sum}} = \text{tr}(E\{\hat{d} - d)(\hat{d} - d)^H\}) = \text{tr}((UHB - I)(UHB - I)^H) + \sigma_n^2\text{tr}(UU^H),
$$

while the MSE of UE $k$ is given by

$$
\epsilon_k = \text{tr}(E\{d_k - \hat{d}_k)(d_k - \hat{d}_k)^H\}) = \text{tr}((U_kH_kB - \Theta_k)(U_kH_kB - \Theta_k)^H) + \sigma_n^2\text{tr}(U_kU_k^H),
$$

where $\Theta_k = [0_{U_k \times \sum_{j=k+1}^{M} U_j}I_{U_k\times \sum_{j=k+1}^{M} U_j}]$. With the sum MSE target $\epsilon_{\text{sum}}$ and the MSE target $\epsilon_k$ of UE $k$, we define the four problems:

P1: Minimizing the sum MSE

$$
\{U^*, B^*\} = \arg\min_{U,B} \epsilon_{\text{sum}} \quad \text{s.t.} \quad \text{tr}(B_mB_m^H) \leq \rho_m \quad \forall m
$$

P2: Minimizing the overall power with target sum MSE

$$
\{U^*, B^*\} = \arg\min_{U,B} \text{tr}BB^H \quad \epsilon_{\text{sum}} \leq \xi_{\text{sum}} \quad \text{s.t.} \quad \text{tr}(B_mB_m^H) \leq \rho_m \quad \forall m
$$

P3: Minimizing the weighted user MSE

$$
\{U^*, B^*\} = \arg\min_{U,B} \epsilon_k/\xi_k \quad \forall k \quad \text{s.t.} \quad \text{tr}(B_mB_m^H) \leq \rho_m \quad \forall m
$$

P4: Minimizing the overall power with user MSE targets

$$
\{U^*, B^*\} = \arg\min_{U,B} \text{tr}BB^H \quad \epsilon_k \leq \xi_k \quad \forall k \quad \text{s.t.} \quad \text{tr}(B_mB_m^H) \leq \rho_m \quad \forall m
$$

Note, that for all problems the partitioning can capture arbitrary groups of antennas and is not restricted to per-BS power constraints.

III. Filter Optimization

A solution for problem P1-P4 considering perfect CSI was presented in [10], where the authors proposed an algorithm that alternately optimizes $B$ and $U$. However, global optimality is not necessarily achieved.

A. Perfect CSI

Considering a fixed precoding matrix the receive filter of UE $k$ minimizing its MSE (11) is

$$
U_k^* = (H_kBB^H\hat{H}_k^H + \sigma_n^2I)^{-1}\hat{B}_k\hat{H}_k^H \quad \forall k.
$$

Computing (16) for all $K$ UEs the precoding matrix $B$ is optimized by solving a second order cone program (SOCP) related to the problem P1-P4, where the SOCP for problem P1 is

$$
B^* = \arg\min_{B} t \quad \text{s.t.} \quad \vec{(UHB - I)} \leq t \quad (17)
$$

A solution can be efficiently found by standard SOCP solvers like SEDUMI [18]. For the according SOCP formulations of problem P2-P4 we refer to [10].

B. Imperfect CSI

In this section we present a robust solution for problem P1-P4 by incorporating statistical knowledge of the channel uncertainty into the filter design. Inserting the channel matrix (2) into (10) results in

$$
\epsilon_{\text{sum}} = \text{tr}((UHB - I)(UHB - I)^H) + \text{tr}(E\{UEBB^HE^H\}) + \sigma_n^2\text{tr}(UU^H),
$$

since $E\{\text{tr}(UHB)\} = 0$, $E\{\text{tr}((UEB)(UBH)^H)\} = 0$ and $E\{\text{tr}(UHB)\} = \text{tr}(UBH)$, according to the results of Section II-B. Again, alternating filter optimization is applied, as described in Section III-A. With a fixed precoding matrix $B$, the robust receive filters are obtained by

$$
U_k^* = (\hat{H}_kBB^H\hat{H}_k^H + \Phi_k + \sigma_n^2I)^{-1}\hat{B}_k\hat{H}_k^H \quad \forall k,
$$

where an additional regularization is achieved by introducing $\Phi_k = \text{vec}(\Sigma_k\text{vec}(BB^H))$ with

$$
\Sigma_k = [\sigma_n^2[k, 1]I_{U_k \times B_1}, \ldots, \sigma_n^2[k, M]I_{U_k \times B_M}].
$$

With fixed receive filters the processing block matrix $B$ is optimized by solving an SOCP again. The additional term in (18) arising from channel uncertainty can be rewritten as

$$
\text{tr}(E\{UEBB^HE^H\}) = ||\Psi_k\vec{B}||_2^2, \quad (20)
$$

which can be integrated into an expanded SOCP. The block diagonal matrix $\Psi = I_U \otimes \text{blkdiag}\{I_{B_1}, \ldots, q_{M}\}$, where each vector $q_m$ is composed of $q_m = \text{vec}([\sigma_n[1, m]U_1, \ldots, \sigma_n[K, m]U_K])$. Note, that in practice receive filter are typically computed at the UEs based on precoded pilots. However, due to the alternating approach, (18) is still required for precoder optimization. The UE specific MSE is obtained by inserting (2) into (11):

$$
\epsilon_k = \text{tr}((U_k\hat{H}_kB - \Theta_k)(U_k\hat{H}_kB - \Theta_k)^H) + \text{tr}(E\{U_kE_kBB^HE_k^H\}) + \sigma_n^2\text{tr}(U_kU_k^H) \quad (21)
$$

According to (20) we can write

$$
\text{tr}(E\{U_kE_kBB^HE_k^H\}) = ||\Psi_k\vec{B}||_2^2 \quad (22)
$$

where $\Psi_k = I_U \otimes \text{blkdiag}\{I_{B_1} \otimes \text{vec}(\sigma_n[k, 1]U_k), \ldots, I_{B_M} \otimes \text{vec}(\sigma_n[k, M]U_k)\}$.
With that result we can adapt the original SOCPs for calculating the precoding matrix $B$ to:

**P1: Minimizing the sum MSE**

$$B^* = \arg\min_{\mathbf{v}} t \quad \text{s.t.} \quad \left\| \begin{array}{c} \psi \text{vec}(\mathbf{B}) \\ \sigma_n \sqrt{\text{tr}(\mathbf{UU}^H)} \end{array} \right\|_2 \leq t, \quad (23)$$

$$\left\| \vec{\mathbf{B}}_m \right\|_2 \leq \sqrt{\rho_m} \quad \forall m$$

**P2: Minimizing the overall power with target sum MSE**

$$B^* = \arg\min_{\mathbf{v}} t \quad \text{s.t.} \quad \left\| \vec{\mathbf{B}} \right\|_2 \leq t \quad \left\| \begin{array}{c} \psi \text{vec}(\mathbf{B}) \\ \sigma_n \sqrt{\text{tr}(\mathbf{UU}^H)} \end{array} \right\|_2 \leq \xi_{\text{sum}}, \quad (24)$$

$$\left\| \vec{\mathbf{B}}_m \right\|_2 \leq \sqrt{\rho_m} \quad \forall m$$

**P3: Minimizing the weighted user MSE**

$$B^* = \arg\min_{\mathbf{v}} t \quad \text{s.t.} \quad \left\| \begin{array}{c} \psi_k \text{vec}(\mathbf{B}) \\ \sigma_n \sqrt{\text{tr}(\mathbf{U}_k\mathbf{U}_k^H)} \end{array} \right\|_2 \leq t/\xi_k \quad \forall k$$

$$\left\| \vec{\mathbf{B}}_m \right\|_2 \leq \sqrt{\rho_m} \quad \forall m$$

**P4: Minimizing the overall power with user MSE targets**

$$B^* = \arg\min_{\mathbf{v}} t \quad \text{s.t.} \quad \left\| \vec{\mathbf{B}} \right\|_2 \leq t \quad \left\| \begin{array}{c} \psi_k \text{vec}(\mathbf{B}) \\ \sigma_n \sqrt{\text{tr}(\mathbf{U}_k\mathbf{U}_k^H)} \end{array} \right\|_2 \leq \xi_k \quad \forall k$$

$$\left\| \vec{\mathbf{B}}_m \right\|_2 \leq \sqrt{\rho_m} \quad \forall m$$

Note, that integrating $\psi \in \mathbb{C}^{|U|B \sum_{k=1}^K U_k^2 \times |U|B}$ results in an increase in computational complexity compared to the non-robust solution.

### IV. Simulation Results

In this section we present results obtained by Monte-Carlo simulations averaged over multiple channel realizations. The applied parameter setup is listed in Table I. The MSE over $\text{SNR} = 10 \log_{10}(\rho/\sigma_n^2)$ for Algorithm P1 is shown in Fig. 2. The black line refers to precoding with perfect CSI. In this case, the proposed solution equals the solution presented in [10] as well as in [19] (by setting the channel uncertainty to zero). As soon as CSI is only imperfectly available for precoding, the proposed robust scheme (green curve) outperforms the solution in [10] (blue curve), since additional statistical knowledge of the channel uncertainty is exploited for designing the filter. In contrast to this work, the robust algorithms presented in [19] for the problems P1-P4 are designed under the assumption of bounded channel uncertainty. However, estimation and quantization of Gaussian channels result in unbounded channel uncertainty, as presented in Section II-B. Therefore, the algorithms from [19] are not appropriately applicable to the considered problems. Robust solutions for unbounded channel uncertainty but sum power constraints [12] can also not be applied directly to the problem. However, appropriate scaling of the resulting precoding matrix designed under sum power constraints could offer an adequate suboptimal solution. However, such comparison is beyond the scope of this paper and is not further discussed. For imperfect CSI the sum MSE of the non-robust solution from [10] only falls until an SNR of 24 dB is reached. For higher SNR values the MSE is increasing again. This behavior occurs since in the moderate SNR regime the noise regularization of the pseudo inverse also compensates for CSI imperfectness. However, with increasing SNR the regularization disappears and the algorithm would converge to the zero-forcing solution. However, with the proposed robust precoding design performance gains can be achieved by integrating statistical

![Fig. 2. Sum MSE by solving P1 for perfect and imperfect (channel estimation and quantization) CSI. The robust solution exploits statistical knowledge of the channel uncertainty which is not considered for non-robust processing.](image-url)
knowledge of the channel uncertainty into the filter design (as presented in Section III-A). Fig. 3 shows the transmit power at the BSs in relation to the power limit. The dashed line is obtained by averaging over the BS with maximum transmit power. Interestingly, at SNR of 24 dB the proposed robust solution does not exploit the maximum allowed transmit power at both BSs. Hence, at high SNRs the MSE increase of the non-robust solution is partially compensated by reducing the transmit power. Results for P2 are shown in Fig. 4. From the left plot it is shown that an MSE target of 0.9 (black line) can be achieved on average (50%) using the robust design, while only 6% fulfill this demand without knowledge of the channel uncertainty. On the contrary, the proposed design has a lower the probability that the problem is feasible (the target MSE can be achieved with the available transmit power) and a larger power consumption (right plot of Fig. 4). Analogous results are achieved for P3 and P4. Equally to [10] dependent on the initialization the presented designs do not necessarily achieve the global optimum.

V. CONCLUSIONS

We presented a robust filter design for linear precoding considering four different MSE related problem formulations with general power constraints. Applying the introduced feedback model capturing unbounded channel uncertainty due to channel estimation and quantization errors performance gains can be achieved compared to existing solutions for the stated problems.

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