Analyzing the Signal-to-Noise Ratio of Direct Sampling Receivers

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Abstract—The increasing demand for multi-mode multi-band operation in mobile communications requires flexible radio frontends. Direct sampling receivers are very promising for this purpose. For this class of receivers, a proper parameterization of the analog-to-digital converter, in terms of input bandwidth, sampling frequency, and quantization resolution, is essential for its operation.

This paper evaluates prospects and challenges of direct sampling receivers analytically. The impact of the sampling frequency and the quantization resolution on the signal-to-noise ratio of the received bandpass signal is approximated with a closed-form analytical expression. This is used to compare the performance of direct sampling receivers to the performance of traditional homodyne receiver concepts. Furthermore, an optimal choice and the trade-off between sampling rate and quantization resolution are discussed for direct sampling receivers that are intended for the reception of LTE signals.

I. INTRODUCTION

Today, many mobile devices/handsets support a multitude of wireless communication standards, e.g., cellular, digital radio and television broadcasting, GPS, WIFI, and Bluetooth. However, multi-mode or multi-band operation is typically only achieved at the cost of increased complexity of the analog and digital frontends. The simplest, but worst receiver design in terms of size and costs is to implement parallel structures. To overcome complexity and cost issues, it is wise to reuse key processing components such as amplifiers, multi-band filters, and down-conversion mixers. Thus, it will be possible to limit the size, costs, and power consumption of the receiver.

Traditionally, homodyne and heterodyne receiver architectures have always been applied for single-band signal reception (see Fig.1(A)). Technological advancements have enabled tunable processing elements, which already allow a limited reconfiguration/adaptation of these receivers. A state-of-the-art overview on implementation techniques and architectures for multi-mode multi-band transceivers is given in [1].

The next step towards flexible radio frontends, which follows Mitolas vision of completely software-defined radio [2], is to shift individual operations, as for example the down-conversion stage of a receiver, from the analog to the digital domain. In this case, the receive signal is sampled in bandpass domain, directly. Such a receiver architecture belongs to the class of direct sampling receivers (DSR), as shown in Fig.1(B). Some DSRs have already been designed for existing mobile communications standards. One example is the GSM/GPRS transceiver frontend presented in [3], which is a single chip solution using 90nm CMOS technology.

To adopt the DSR architecture more widely, its performance in terms of the effective signal-to-noise ratio (SNR) of the received signal samples have to be studied in detail. There are two parameters that have a significant impact on the quality of the received signal beside the receiver input bandwidth. These are the applied sampling frequency $f_{sub}$ and the quantization resolution $b$. Both have to be adjusted carefully in order to meet the SNR requirements for proper signal reception.

To model the SNR, the common approach is to consider the received bandpass signal with fixed bandwidth $B_0$ at its respective carrier frequency $f_c$. This bandpass signal is directly sampled at the receiver with a rate $2B_0 < f_s < f_c$. This inherently translates the signal to an intermediate frequency lower than the actual carrier frequency. This is termed sub-sampling and the underlying theory can be found, e.g., in [4]. One of the drawbacks of sub-sampling is that it introduces an increased noise density in the down-converted frequency band due to the effect of noise folding [4]. The discretization of the received signal also includes amplitude quantization. The theory of quantization and quantization schemes for minimum distortions can be found in [5] and [6]. To evaluate the performance of the DSR, a detailed analysis of the effective SNR that results for a given sampling rate and quantization resolution is necessary.

![Fig. 1: Block diagram of a (A) Direct Conversion Receiver and a (B) Direct Sampling Receiver. All components with light gray background are digital.](image-url)
To the best knowledge of the authors, this has not been considered in literature, so far. Hence, this paper provides an analysis of sampling rates and quantization resolutions to be used in DSRs to satisfy SNR requirements. It studies the properties of the effective in-band SNR $\gamma_{\text{eff}}$ by deriving an analytical closed-form expression depending on the actual SNR $\gamma_{\text{in}}$ of the received bandpass signal as well as on the receiver input bandwidth $f_{\text{AFE}}$, sampling frequency $f_s$, and the quantization resolution $b$. It will be shown that the DSR architecture is feasible for various applications even with a finite quantization resolution and reasonable sub-sampling frequencies. Furthermore, some insight into the proper choice of sampling frequency and quantization resolution for a given SNR $\gamma_{\text{in}}$ and maximum performance loss are provided. The paper also provides a practical evaluation of DSRs for the reception of LTE signals.

The remainder of this paper is organized as follows: Section II introduces the system model and basic equations used to derive the expression of the effective SNR $\gamma_{\text{eff}}$. Section III considers the effect of noise shaping for the novel case of low quantization resolution and shows the integration in the known model. A proper choice of the sub-sampling frequency in combination with the quantization resolution for a given system performance is derived in Section IV. The application to a LTE system in Section V explains interrelations between the sub-sampling frequency and the quantization resolution for a certain maximum loss in the SNR of the received signal. Finally, Section VI summarizes the content of the paper.

II. SYSTEM MODEL

The considered model of the DSR is depicted in Figure 2. The band-limited signal $s(t)$ has a cutoff frequency $f_0$, bandwidth $B_0 = 2f_0$, and is received at the carrier frequency $f_c$. $n(t)$ models the thermal noise characteristics of the receiver as additive white Gaussian noise (AWGN) with zero mean and variance $\sigma_n^2$. Both, the received signal and the noise are assumed to have a uniform power spectral density but with different bandwidths.

Now, we will discuss the sampling and the quantization process of the A/D converter in Fig. 2 in detail. The sampling unit of the A/D converter consists of two elements: the sample-and-hold circuit (S&H) and the quantizer.

The S&H is modeled as a combination of a filter $h_{\text{ADC}}(t)$ and an ideal sampling circuit (switch). The filter impulse response is assumed to have a rectangular shape of length $T_A$, where $T_A$ corresponds to the aperture time of the S&H. The rate of the switch corresponds to the sampling time $T_s$. It has to be at least twice as high as the highest frequency $f_{\text{Nyq}} = 2 \cdot (f_c + f_0)$ according to Nyquist’s theorem [7], [8]. For simplicity, we will assume that $f_{\text{Nyq}} = 2f_{\text{AFE}}$ which fulfills the Nyquist’s theorem.

When applying sub-sampling, which is often also referred to as under-sampling or direct bandpass sampling, the sampling frequency $f_s$ has to be in the range $2B_0 \leq f_s < f_{\text{Nyq}}$ and chosen properly to replicate the signal spectrum in the first Nyquist region without introducing self-aliasing [4].

To compare sub-sampled signals to Nyquist sampled signals, we define a sub-sampling factor as

$$F_{\text{sub}} = \frac{f_{\text{Nyq}}}{f_s} = 2\frac{f_{\text{AFE}}}{f_s} \geq 1.$$  \hspace{1cm} (1)

The factor will be used to analyze the effective SNR of the received signal samples $x[n]$ in general without considering specific sampling frequencies. It has been shown in [4] that the SNR $\gamma_{\text{sub}}$ after sub-sampling degrades proportional with $\frac{1}{F_{\text{sub}}}$ due to noise folding from higher Nyquist regions. Hence, the SNR of $x[n]$ as a function of the in-band SNR $\gamma_{\text{in}}$ and $F_{\text{sub}}$ can be expressed as

$$\gamma_{\text{sub}}(\gamma_{\text{in}}, F_{\text{sub}}) = \frac{\gamma_{\text{in}}}{F_{\text{sub}}}$$  \hspace{1cm} (2)

for positive integer sub-sampling factors $\forall F_{\text{sub}} \in \mathbb{N}^+$. It is important to note that sub-sampling changes the absolute value of the spectral power density of the noise proportionally with the sub-sampling factor while the overall noise power remains constant.

Let us now also consider the effect of the quantization of the received signal samples. A well-known analytical model of a uniform symmetric quantizer, describing the influence of quantization distortion $q[n]$, is the pseudo quantization noise model from [5]. It defines the signal-to-quantization-noise ratio (SQNR) $\gamma_q(b)$ as a function of the quantization resolution $b$ in bit ($M = 2^b$ steps). The average power of the error signal $q[n]$ is considered to be $E_q[|q|^2] \approx \frac{\Delta s^2}{12}$ under the assumption of full-scale input samples $x[n]$, where $\Delta s$ is the quantization step size, and $E_q[|a|^2]$ denotes expectation of $a$ in terms of $x$. Furthermore, the spectral density of the quantization noise is assumed to be uniform in the interval $[-\frac{\Delta s}{2}, \frac{\Delta s}{2}]$. The resulting quantization noise power $E_q[|q|^2]$ can be reduced if the sampling rate $f_s \gg 2f_0$ is higher than twice the cutoff frequency of the input signal $s(t)$ and additional digital filtering is applied to the quantized signal samples. By increasing the sub-sampling factor $F_{\text{sub}}$, the oversampling ratio (OSR), which is defined

![Fig. 2: System model of the DSR.](image-url)
as the ratio of half the sampling frequency to the signal bandwidth, is affected inversely proportional:

\[ \text{OSR} = \frac{f_{\text{nyq}}}{4f_0} \cdot \frac{f_{\text{AFE}}}{2f_0}. \]  

(3)

Let \( \eta_x \) denote the peak-to-average-power ratio (PAPR) of the unquantized signal samples \( x[n] \), which is defined as \( \eta_x = \frac{\max|x|^2}{E_x[|x|^2]} \) with \( \max|x| = 2^b \Delta x \). This leads to an average power of \( E_x[|x|^2] = \left( \frac{2^b \Delta x}{\eta_x} \right)^2 \). The resulting SQNR of the quantized received signal samples can be derived as

\[ \gamma_q(b, \text{OSR}, \eta_x) = \frac{E_x[|x|^2]}{E_y[|y|^2]} = \frac{3 \cdot 4^b}{\eta_x \cdot \text{OSR}}. \]  

(4)

Combining the derivations of \( \gamma_{\text{sub}}(\gamma_{\text{in}}, \text{OSR}) \) and \( \gamma_q(b, \text{OSR}, \eta_x) \) allows to formulate the effective in-band SNR \( \gamma_{\text{eff}} \) of the digital receive signal \( y[n] \) of the DSR as

\[ \gamma_{\text{eff}}(\gamma_{\text{in}}, F_{\text{sub}}, b, \text{OSR}) = \frac{\gamma_q \gamma_{\text{sub}}}{\gamma_q + \gamma_{\text{sub}} + 1} \]  

(5)

\[ = \frac{F_{\text{sub}} + \frac{\eta_x}{3 \cdot 4^b \cdot \text{OSR}} (F_{\text{sub}} + \gamma_{\text{in}}) \} - \gamma_{\text{in}}. \]

Finally, we can find a closed-form expression for the effective in-band SNR of a DSR:

\[ \gamma_{\text{eff}}(\gamma_{\text{in}}, F_{\text{sub}}, b, \eta_x) = \frac{\gamma_{\text{in}}}{F_{\text{sub}} \left(1 + \frac{2 \gamma_{\text{in}}}{\eta_x} \frac{F_{\text{sub}}}{3 \cdot 4^b \cdot \text{OSR}} \right)}. \]  

(6)

The equation illustrates that sub-sampling and quantization are the two main reasons for an SNR degradation. At the one hand, higher sub-sampling factors, corresponding to lower sampling frequencies, result in a degraded SNR. On the other hand, lower quantization resolution also affects the SNR in a negative way. One way to improve the effective SNR is a higher OSR. The analytical expression is used to predict the SNR in further derivations.

III. ADJUSTING THE EFFECTIVE SNR FOR LOW QUANTIZATION RESOLUTION

In Section II above, a uniform power spectral density (PSD) has been assumed for the quantization error. This is only exact for high resolution A/D converters [5]. However, the PSD of the error signal is different if the quantization resolution is low. From Fig.3 can be observed, that the quantization noise is concentrated (shaped) around the signal-of-interest in frequency domain. This noise shaping becomes noticeably smoother for an increasing resolution \( b \).

To account for the shaped quantization noise, the mean in-band quantization noise power \( E_y[|y|^2] \) has to adapted for the low resolution case. For this purpose, we introduce the scaling factor \( \zeta \) which is a function of the quantization resolution \( b \) and the OSR.

Hence, (4) is extended by \( \zeta \) as follows:

\[ \gamma_q(b, \text{OSR}, \eta_x, \zeta) = \frac{1}{\zeta} \cdot \frac{3 \cdot 4^b}{\eta_x \cdot \text{OSR}}. \]  

(7)

The scaling factor \( \zeta \) can be obtained empirically by computing the ratio of the effective in-band SNR given in (6) and the true effective in-band SNR that results from simulations, as detailed in the following.

A. Calculation of the Scaling Factor \( \zeta \)

The simulation considers band-limited normally distributed samples \( x \sim \mathcal{N}(0, 1) \) which are the output to a uniform symmetric quantizer with a resolution of \( b \) bits. The output values \( y \) and the decision levels of the quantizer are determined according to the minimum mean-square error (MMSE) criterion for minimum distortion [6]. The time-domain error signal \( e = y - ax \) is calculated by using BUSSGANGs theorem [9]. \( \alpha \) can be pre-computed as shown in [10].

The simulated SNR can now be determined by calculating the ratio of the PSDs of the output signal \( S_{yy}(f) \) and of the error signal \( S_{ee}(f) \) as follows:

\[ \gamma_{\text{sim}}(f) = \frac{S_{yy}(f) - S_{ee}(f)}{S_{ee}(f)} \quad f < f_0 \]  

(8)

For a discrete time simulation, the continuous PSDs have to be approximated by their discrete representations. It suffices to calculate the mean in-band SNR only for frequencies smaller than \( f_0 \) (cutoff frequency of \( x[n] \)). \( m \) is considered to be the discrete frequency index. Finally, the scaling factor \( \zeta \) can be expressed as a function of OSR and quantization resolution \( b \):

\[ \zeta(b, \text{OSR}, \eta_x) = \frac{E_m[\gamma_{\text{sim}}(m)]}{\text{OSR} \cdot \text{OSR}}. \]  

(9)

B. The Adjusted Effective SNR

The obtained values for the scaling factor \( \zeta \) and the corresponding change in the effective SNR are shown in Fig. 4 and Fig. 5, respectively. The scaling factor decreases monotonically while increasing quantization resolution, which holds for any OSR. Furthermore, it can be observed that the scaling factor is the smaller the OSR is. When applying the scaling factor to the computation of the effective SNR of the DSR its accuracy can be improved. The scaled SNR given in (7) provides results that are very close to the results

Fig. 3: Quantization noise shaping of band-limited signals for low quantization resolution, i.e., \( b \in [1, 5] \) bit and OSR = 4.
obtained from a simulated receiver chain with exactly modeled low and high quantization resolutions. Note however, that the derivation of an analytical SNR expression that accounts for the quantization error shaping as a function of the OSR, quantization resolution, and the carrier frequency remains a topic for further research.

IV. PROPER CHOICE OF THE SUB-SAMPLING FREQUENCY AND QUANTIZATION RESOLUTION

After dealing with the analytical derivation of the effective in-band SNR \( \gamma_{\text{eff}} \) of the DSR, we will now ask another important question: How to choose the sub-sampling frequency and the quantization resolution for a desired system performance? This question is highly motivated by the fact that practical receivers are typically designed under certain constraints of the input SNR and maximum noise figures.

To give an answer to this question, the term \( \alpha \), which describes the maximum loss in the SNR between input and output of the sub-sampling A/D converter, is introduced:

\[
\alpha = \frac{\gamma_{\text{eff}}}{\gamma_{\text{in}}} = \frac{1}{F_{\text{sub}} \left(1 + \frac{2f_{\text{in}}}{4f_{\text{AFE}}(F_{\text{sub}} + \gamma_{\text{in}})}\right)} < 1
\]  

We can refer to \( \alpha \) as noise figure of the sub-sampling A/D converter. (10) shall now be used to derive a expression for the sub-sampling factor \( F_{\text{sub}}(b, \alpha, \gamma_{\text{in}}) \) as a function of the quantization resolution \( b \), the allowed A/D converter noise figure \( \alpha \) and the in-band SNR \( \gamma_{\text{in}} \) at the input. The behavior of \( F_{\text{sub}} \) is depicted in Fig. 6 for two different values of \( \alpha = \{-3, -10\} \) dB and \( \gamma_{\text{in}} = 50 \) dB.

The maximum sub-sampling factor \( F_{\text{sub}} \) that can be allowed in case of high quantization resolution is always limited by \( \alpha \). In this case, the SNR loss is dominated by the noise folding effect. However, if the quantization resolution is low, the quantization error strongly influences the effective SNR of the received signal samples. Hence, feasible sub-sampling factors have to be chosen much lower to avoid high degradations of the SNR. The difference between analytical and simulation results that can be observed in Fig.5 for low quantization resolution originate from approximation of the quantization error as introduced in Sec.III.

A proper choice of the parameters \( F_{\text{sub}} \) and \( b \) for a DSR with given input SNR \( \gamma_{\text{in}} \) and limited A/D converter noise figure \( \alpha = -3 \) dB, is given by the gray area below the plotted curve. The two parameters \( F_{\text{sub}} \) and \( b \) directly translate into feasible configurations of the sampling frequency and quantization resolution of the DSR.
V. EXAMPLE APPLICATION: A DSR FOR LTE

One promising application for DSRs is the reception of LTE signals in future mobile handsets. The LTE standard allows inter- and intra-band carrier-aggregation (CA) for dynamic data rate adaptation. This requires receiver designs for a parallel reception of multiple contiguous or non-contiguous frequency bands. To overcome the classical approach of duplicating the components of the analog receiver front-end, the DSR appears to be a feasible architecture while sustaining the necessary link performance in terms of SNR. This section analyzes the prospects and challenges of DSRs for LTE.

Table I summarizes the allocated bands for inter- and intra-band CA in the LTE downlink, as defined in the LTE standard 3GPP TS 36.101. Now, consider that Band 1, 5, and 40 are received simultaneously. To obtain a contiguous band configuration in the first Nyquist zone, the sampling frequency has to be $f_s = 383$ MHz.

![Table I: Downlink operating bands for LTE-CA specified in 3GPP TS 36.101.](image)

For the following evaluations, we have picked the Band 40 which is defined for intra-band CA, and specified the signal bandwidth and carrier frequency to $B_0 = 20$ MHz and $f_c = 2310$ MHz. The Nyquist sampling frequency is $f_{nyq} = 4800$ MHz. Further parameters are specified in Table II.

![Table II: Configuration of the simulation chain.](image)

The behavior of the effective in-band SNR $\gamma_{\text{eff}}$ is shown in Fig. 5, above for $\gamma_{\text{in}} = 50$ dB. Considering low quantization resolution, the effective in-band SNR is limited by the quantization resolution $b$ and the OSR. Different sub-sampling frequencies have a negligible impact on the SNR. When we increase the quantization resolution, the effect of noise folding interacts with the SNR limitation due to quantization resolution. For $b \to \infty$, the depicted curves are approaching their maximum value which is defined by the SNR $\gamma_{\text{in}}$. Here, the dominant impact of noise folding is obvious as the effective SNR is decreasing by 3 dB when doubling the sub-sampling factor.

The effect of noise folding is one of the main limitations of DSRs which always degrades the overall performance as compared to a direct conversion receiver. For our exemplary LTE scenario, the lowest sub-sampling frequency is determined by $f_{\text{sub}} = 2 \cdot B_{\text{Band 40}} = 200$ MHz. This results in a sub-sampling factor of $F_{\text{sub}} = \frac{f_{\text{sub}}}{f_c} = 24$ according to (1), and increases the SNR by $10 \log_{10} 24 \approx 14$ dB. This value can be interpreted as the corresponding noise figure of the sub-sampling ADC for high quantization resolution.

If the DSR is designed for a mobile device, the power consumption is another important figure of merit. In this case, the trade-off between the consumed energy and the performance of the receiver has to be studied carefully.

In the end, the sub-sampling frequency has to be carefully chosen in conjunction with the quantization resolution and the power consumption to fulfill required systems specifications.

VI. SUMMARY

The DSR architecture appears to be a promising architecture for signal reception without using an analog down-conversion mixer at all. However, the prospects and challenges of this architecture have to be evaluated carefully.

This paper has analyzed the effective SNR of DSRs. It has provided closed-form SNR expressions as a function of the input bandwidth of the receiver, as well as of the sampling frequency and the quantization resolution. In order to calculate realistic SNR values for low quantization resolution, a scaling factor $\zeta$ has been introduced. The factor ensures a consistent behavior of the simulated and analytical results, and aligns the quantization noise according to the observed values. Furthermore, the proper choice of sampling frequency and quantization resolution for fixed values of the in-band SNR of the received bandpass signal and given values of the maximum SNR loss due to the receiver have been discussed in more detail. It has been shown that the solution space can be determined by a formal relationship between the sub-sampling factor as a function of input SNR, the quantization resolution, as well as the maximum SNR loss.

REFERENCES