Low Peak-to-Average Power Ratio for Next Generation Cellular Systems with Generalized Frequency Division Multiplexing

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Abstract—With the Internet of Things on the horizon, novel services will impose new challenges upon future cellular systems. One key parameter that influences the energy efficiency is peak-to-average power ratio (PAPR). PAPR is particularly relevant for cheap mass market terminals. In the most recent cellular standard, Long Term Evolution, different waveforms are used in downlink and uplink transmission, to address the issue of non-linear distortions as a consequence of high PAPR. In this work, generalized frequency division multiplexing (GFDM) is explored as a possible, non-orthogonal waveform for next generation cellular systems. Particular focus lies on the PAPR properties of the generated waveform.

First, various filtered multi-carrier techniques, including OFDM, SC-FDE and GFDM, are presented in an unified framework. Second, the PAPR of those schemes is compared in an uplink multiple access scenario. The results show that a non-orthogonal waveform brings additional degrees of freedom, which can be beneficial.

Index Terms—filtered multi-carrier techniques, OFDM, SC-FDE, SC-FDMA, GFDM, PAPR

I. INTRODUCTION

Peak-to-average power ratio (PAPR) is a key performance parameter of wireless communication schemes that impacts cost and energy efficiency of the hardware equipment. A high PAPR means that linear amplifiers with large input backoff need to be used, in order to avoid distortions in the transmitted signal, otherwise bit error rates of the transmission can increase and strong out-of-band radiation may be introduced. The input backoff is especially relevant for the battery lifetime of mobile terminals, as it impacts the energy consumption of a device, while another issue is heat dissipation of the circuitry. Particularly in multi-carrier systems, high PAPR is an issue, because the individual sub-carrier signals can easily add up in a constructive way and produce high signal amplitudes. In the current generation of cellular systems, Long Term Evolution (LTE), orthogonal frequency division multiplexing (OFDM) is used for downlink transmission. Various techniques for PAPR reduction have been investigated [1] for this scheme during the past years. They however, are not in the scope of this work. To alleviate the problem of high PAPR for mobile terminals, single-carrier frequency division multiple access (SC-FDMA) has been introduced in the uplink. With SC-FDMA, each individual user transmits a single-carrier signal and that is of benefit for the peak values in the signal [2]. Beyond LTE, new applications and use cases are envisioned for next generation cellular systems. For instance, the trend of the ”Internet of Things” predicts that a single subscriber today will own a number of wireless connected sensors and devices in a few years. This kind of machine type communication (MTC) will bring a number of new requirements and challenges for mobile operators. The ability to handle magnitudes of higher number of subscribers is especially relevant for MTC, where the vision is to enable cheap terminals that require little maintenance and a battery lifetime of several years for mass market. One approach to address these requirements in the future can be a 5G cellular system based on a new waveform that is designed with particular focus on energy efficiency and consequently on PAPR. Generalized frequency division multiplexing (GFDM) [3] has been proposed as a new waveform for 5G. GFDM is a non-orthogonal, block-based multi-carrier modulation scheme with circular signal properties in time and frequency domain, which can be efficiently implemented based on the fast Fourier transform (FFT) algorithm [4].

The remainder of this paper organized as follows: In Section II, it is shown how GFDM relates to other well known transmission schemes, including OFDM. Section III presents the setup in which the PAPR performance of these schemes is evaluated. The results of the simulations are discussed in Section IV. A summary is given in Section V.

II. SYSTEM OVERVIEW

Consider a general transmitter model according to Fig. 1. Data from a binary source is first mapped to a set of complex constellation points, e.g. with quadrature amplitude modulation (QAM), producing the data symbols $d$. Depending on the transmit scheme, $d$ undergoes digital baseband processing steps, which yield the transmit samples $x$. In the next step, a cyclic prefix (CP) is added to the signal, producing $\tilde{x}$. Then a baseband transmit filter is applied, which converts the complex samples in $\tilde{x}$ to a waveform $\tilde{x}$. What follows in the chain are analog components like digital-to-analog converter (DAC), mixer, power amplifier and antennas. Note that the aforementioned baseband transmit filter could also be applied in the analog domain as part of the DAC. However, in this
work, it is modeled as an oversampled digital filter. In the next sections, the digital baseband processing of different schemes will be discussed.

**A. OFDM**

Let \( \mathbf{d} = (d_0, \ldots, d_{N-1})^T \) be a complex vector containing \( N \) data symbols \( d_n \). Fig. 2(a) shows the mapping of such a vector to time and frequency resources in OFDM, exemplary for \( N = 8 \). Given a block duration \( T \), the signal consists of \( N \) sub-carriers which are spaced \( \Delta f = \frac{1}{T} \) apart. The signal occupies a total bandwidth of \( B = \frac{N}{T} \). Each sub-carrier consists of \( N \) time samples which are \( \Delta t = \frac{T}{N} \) apart. The transmit samples \( \mathbf{x} \) can be obtained as

\[
\mathbf{x} = \mathbf{W}_N^H \mathbf{d},
\]

where \( \mathbf{W}_N = (w_{ij})_{N \times N} \) is a discrete Fourier transform (DFT) matrix with \( w_{ij} = e^{-j2\pi ij/N} \) and \( i, j \in \{0, \ldots, N-1\} \). OFDM is a wide spread multi-carrier scheme that has become part of various wireline and wireless standards, including DSL, IEEE 802.11, DVB-T and LTE. A major advantage that makes it a favored candidate is the fact that, in a multi-path fading scenario, the wireless channel is divided into frequency flat sub-channels. In combination with a cyclic prefix, this makes it a favored candidate is the fact that, in a multi-path fading scenario, the wireless channel is divided into frequency flat sub-channels. In combination with a cyclic prefix, this can be of additional benefit for the PAPR, compared to OFDM. When using the single-carrier scheme in a multipath environment, the channel is not necessarily flat. However, with a cyclic prefix, frequency domain equalization (FDE) can be performed at the receiving side. The scheme is then referred to as SC-FDE.

**B. SC-FDE**

In (1), the data \( \mathbf{d} \) is defined in frequency domain. According to Fig. 2(a), this means that the individual elements \( d_n \) occupy a small bandwidth and a long time duration. The counterpart of this scheme is a single-carrier (SC) transmission, where the data \( \mathbf{d} \) is defined in time domain, so that each \( d_n \) occupies a large bandwidth and a small time duration. The baseband processing of a single-carrier transmission is as simple as

\[
\mathbf{x} = \mathbf{d}
\]

and yields the grid drawn in Fig. 2(b). Here, the subsequent transmit filter shown in Fig. 1 determines the spectral properties of the signal. Depending on the pulse shape, this can be of additional benefit for the PAPR, compared to OFDM. When using the single-carrier scheme in a multipath environment, the channel is not necessarily flat. However, with a cyclic prefix, frequency domain equalization (FDE) can be performed at the receiving side. The scheme is then referred to as SC-FDE.

**C. SC-FDM**

Now consider the combination of the two schemes presented above. For this purpose, the total bandwidth \( B \) is divided into \( K \) sub-carriers and the symbol duration \( T \) is split into \( M \) time-slots. Note that the total number of transmitted data does not change, i.e. \( KM = N \). The time and frequency resources are merely partitioned in a different way, as depicted in Fig. 2(c). The terminology presented in Fig. 2(d) will be used.

To describe the baseband processing in terms of a linear model, let \( \mathbf{d} = (d_0^T, \ldots, d_{K-1}^T)^T \) be a concatenation of \( K \) sequences \( d_k = (d_{0,k}, \ldots, d_{M-1,k})^T \) with \( M \) elements each. Therein, \( d_{0,n}^k \) corresponds to the data symbol transmitted on the \( k \)th sub-carrier and in the \( n \)th time-slot. The transmit signal that corresponds to one block is then obtained as

\[
\mathbf{x} = \mathbf{W}_N^H \sum_{k=0}^{K-1} \mathbf{P}_k \mathbf{W}_M \mathbf{d}_k.
\]

\( \mathbf{W}_M \) is a Fourier matrix which transforms the data vectors \( \mathbf{d}_k \) to frequency domain. Then they are aligned next to each other with a mapping matrix \( \mathbf{P}_k \) for the \( k \)th sub-carrier, which can be constructed according to

\[
\mathbf{P}_0 = (\mathbf{I}_M \ 0_M \ \cdots \ 0_M)^T, \\
\mathbf{P}_1 = (0_M \ \mathbf{I}_M \ 0_M \ \cdots \ 0_M)^T, \\
\vdots \\
\mathbf{P}_{K-1} = (0_M \ \cdots \ 0_M \ \mathbf{I}_M)^T,
\]

with \( 0_M \) being an \( M \times M \) matrix containing zeros and \( \mathbf{I}_M \) denoting the identity matrix of same size. The inverse DFT \( \mathbf{W}_N^H \) brings the signal back to time domain after summation. Undergoing this processing, each sequence \( \mathbf{d}_k \) individually produces a single-carrier signal. As several of those are then stacked along the frequency axis, it becomes a frequency division multiplex (FDM), hence SC-FDM.

This scheme has been adopted in the LTE uplink specifications [5] for multiple access as SC-FDMA, where each sub-carrier is assigned to one user. In contrast to OFDMA, which is used in the downlink, SC-FDMA leads to a reduced PAPR in the uplink.

**D. GFDM**

The difference between GFDM and SC-FDM is in the spectral shaping of the individual sub-carriers. Latter scheme implicitly assumes rect-shaped sub-carriers as depicted in Fig. 3. This restriction is lifted in GFDM, which however means, that the orthogonality between sub-carriers and time-slots in one block can be lost. For instance, one non-Nyquist pulse is the raised cosine (RC) function, where the roll-off parameter...
denotes, how much overlap is produced between neighboring sub-carriers.

The GFDM baseband processing, which includes variable sub-carrier filters, is given by the expression

\[ x = W_H^T \sum_{k=0}^{K-1} P'_k \Gamma R W_M d_k. \tag{5} \]

Here,

\[ \Gamma = \begin{pmatrix} \gamma_0 & 0 & \cdots & 0 \\ 0 & \gamma_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \gamma_{LM-1} \end{pmatrix} \]  

\[ R = \begin{pmatrix} I_M & \cdots & I_M \end{pmatrix}^T. \tag{7} \]

Note that RC filters only require \( L = 2 \). Finally, the mapping matrix \( P'_k \) fulfills the same role as \( P_k \). However it has a slightly different structure, so that the sub-carrier ordering depicted in Fig. 3 is achieved:

\[ P'_0 = \begin{pmatrix} I_{LM/2} & 0_{LM/2} & \cdots & 0_{LM/2} & 0_{LM/2} \end{pmatrix}^T \]

\[ P'_1 = \begin{pmatrix} 0_{LM/2} & I_{LM/2} & \cdots & 0_{LM/2} & 0_{LM/2} \end{pmatrix}^T \]

\[ \vdots \]

\[ P'_{K-1} = \begin{pmatrix} 0_{LM/2} & 0_{LM/2} & \cdots & 0_{LM/2} & I_{LM/2} \end{pmatrix}^T. \tag{8} \]

GFDM can be seen as a general block based multi-carrier concept, which contains OFDM, SC-FDE as well as SC-FDM as a special cases. OFDM is obtained from (5) for \( K = N \) and \( M = 1, L = 1 \) and \( \Gamma = 1 \). Further, the mapping \( P_k \) needs to be used instead of \( P'_k \). To turn (5) in a SC-FDE signal, it is necessary to use the parameters \( K = 1 \) and \( M = N, L = 1, \Gamma = I_N \) and \( P'_k = I_N \). Lastly, (5) becomes equivalent with SC-FDM, when \( L = 1, \Gamma = I_N \) and \( P'_k \) is replaced by the mapping \( P_k \). As shown in Fig. 6, all four schemes are subsets of "filtered multi-carrier Techniques", which have been introduced by [6] and [7] in the sixties of last century.

Introducing non-orthogonal sub-carriers increases the flexibility of the scheme and by this allows to control various properties of the signal. However, one important drawback of non-orthogonality is self-created inter-carrier and inter-symbol interference. One way to mitigate it, is the offset-QAM approach [8]. Another option to deal with this self-interference is successive interference cancellation (SIC). A simple SIC scheme for GFDM has been presented in [9], where introducing some additional complexity to the receiver makes the scheme as good as OFDM in terms of bit-error-rate performance. However, considering that wireless communication systems have to deal with all sorts of impairments anyways, e.g. time and frequency offsets, multi-antenna and multi-user interference, etc., it may be challenging to maintain...
the orthogonality of an OFDM-based signal. Thus interference cancellation algorithms are already part of modern communication systems and the complexity overhead that a non-orthogonal scheme produces in the receiver becomes rather negligible, while the reward of non-orthogonal transmission are additional degrees of freedom. This affects many properties of the signal, including PAPR, which will be discussed in the next section.

III. SIMULATION SETUP

Consider a cellular uplink scenario, which is characterized by several mobile terminals transmitting to a single base station. Multiple access is achieved by frequency division, i.e. a continuous portion of the system bandwidth is reserved for each terminal. For the considerations in this paper, the uplink signal of an individual terminal is analyzed. The schemes that are compared in terms of transmitter peak-to-average power ratio, namely orthogonal frequency division multiple access (OFDMA), single-carrier frequency division multiple access (SC-FDMA) and generalized frequency division multiple access (GFDMA), are depicted in Fig. 4. Here, the matrices $P_k$ and $P'_k$ have the function of scheduling terminals.

For all three schemes, a common IDFT size of $N = 256$ is considered, where one out of $K = 4$ possible terminals is transmitting a data sequence $d_k$ of length $M = 64$. The same baseband transmit filter $G_{tx}$ is used in all schemes. It is a $Q = 8$ times oversampled raised cosine filter with roll-off $\beta = 0.5$. The GFDMA sub-carrier filter $\Gamma$ has also a raised cosine shape. Consequently $L = 2$ sub-carriers overlap in frequency domain. For $\Gamma$, roll-offs $\alpha \in \{0.1, 0.5, 0.9\}$ are considered. The results in the next section represent an average of $10^6$ realizations of transmitted sequences, each having the length of 100 blocks.

IV. SIMULATION RESULTS

The PAPR of a sequence $s = (s_i)^T$ with $i \in \{0, \ldots, N_s - 1\}$ is defined as

$$\text{PAPR} (s) = \max \left( \frac{1}{N_s} \sum_{i=0}^{N_s-1} |s_i|^2 \right).$$

A typical way of visualizing this kind of results is a complementary cumulative density function (CCDF) as in Fig. 5(a), which gives the probability that the PAPR exceeds a certain value $\text{PAPR}_0$ versus $\text{PAPR}_0$. Intuitively, curves further to the left impose weaker requirements to the amplifier, while curves to the right demand a higher linear range and backoff.

From the three schemes in Fig. 4, OFDMA exhibits the worst performance. As previously discussed, this is mainly because there the data is defined in frequency domain and due to the multi-carrier nature of the signal, the orthogonal sub-carrier frequencies can add up constructively and produce high peaks.

In SC-FDMA, the data is defined in time domain, which reduces the number of sub-carriers effectively to one. With that property, the scheme can achieve better performance. For instance, looking at a probability of $10^{-3}$, the SC-FDMA signal has a PAPR which is approximately 2dB lower than OFDMA in this setup. This advantage allows to alleviate the linearity requirements for mobile terminals and is the reason why it used in the LTE uplink. The result is in line with [2]. Finally, the additional degree of freedom from the adjustable sub-carrier filters in GFDMA allows further control of the PAPR. By varying the roll-off of the raised cosine shape from $\alpha = 0.1$ through $\alpha = 0.5$ to $\alpha = 0.9$, additional PAPR reduction of roughly 4dB can be achieved. Note that the results for $\alpha = 0$ would be identical with the SC-FDMA curve. One interesting point is that, looking at the GFDMA sub-carrier filters in time domain in Fig. 5(b), the PAPR of the individual pulse gets worse with increasing roll-off. All three pulses have the same peak, but the the side-lobes of $\alpha = 0.1$ contribute more to the signal average in the denominator of the PAPR than $\alpha = 0.5$ and $\alpha = 0.9$. But when a sequence of several overlapping pulses is transmitted, the bigger side-lobes have a higher chance of adding constructively with the neighboring time-slots, which can produce higher signal peaks. This reflects in the CCDF results, where $\alpha = 0.9$ has a better performance than $\alpha = 0.1$.

Intuitively, the gains in the SC-FDMA and GFDMA can be explained as follows: First, the number of sub-carriers is
Reduced, effectively making it a single-carrier system. Then a pulse shape is applied and gives further control over the signal peak and average. The PAPR performance gain does not come at the cost of the non-orthogonal transmission, which leads to increased bit error rates and requires a more complex receiver [9].

V. SUMMARY

The first contribution of this work is to present an unified framework that shows the relation between the schemes OFDM, SC-FDE, SC-FDM and GFDM. Here, OFDM and SC-FDE are two corner cases. In a first step of generalization, SC-FDM can be seen as a mix of these two. A further degree of freedom is then added with GFDM, by introducing pulse shaping filters for the individual sub-carriers. As shown in Fig. 6, all schemes can be considered as a subset of “filtered multi-carrier techniques”.

The second contribution in the PAPR comparison in an uplink multiple access scenario. Here, OFDMA shows poor performance, because the many sub-carriers can add up constructively and by this produce a high peak power in the signal. SC-FDMA alleviates this problem by transforming the signal to a single-carrier transmission each terminal. In GFDMA, by adjusting the pulse shape that is applied to the sub-carriers, additional gains can be achieved. In a comparable uplink scenario, GFDM can perform even better than SC-FDMA in terms of PAPR.

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