Improved ACLR by Cancellation Carrier Insertion in GFDM Based Cognitive Radios

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Abstract—Generalized Frequency Division Multiplexing (GFDM) is a recent multicarrier modulation technique with low out-of-band radiation that makes it an attractive choice for the PHY layer of cognitive radio. In orthogonal frequency division multiplexing (OFDM) the out of band leakage is around −13 dB; with raised cosine or root raised cosine pulses, the out of band leakage of a GFDM system is around −35 dB. To improve the adjacent channel leakage ratio (ACLR) even further, a technique of inserting cancellation carriers is implemented. Cancellation carriers are inserted at the vicinity of interference avoidance notch and are designed such that these cancellation carriers mitigate the interference from other subcarriers to the adjacent band. With this technique the out of band leakage is lowered to around −65 dB. This stunning improvement in the GFDM adjacent channel leakage ratio satisfies stringent FCC requirements for cognitive radio transmissions in TV white space scenarios.

I. INTRODUCTION

Radio frequency spectrum is a scarce resource in wireless communication. With proliferation of large screen smart wireless devices, innovative services and growing number of mobile users, the huge demand for radio spectrum has made this scarcity more stark. To cope with this, regulatory bodies like Federal Commission for Communication (FCC) in US and Electronic Communications Committee within the European Conference of Postal and Telecommunications Administrations (ECC CEPT) in Europe have recently published some regulatory rules, to make the unused spectrum in the TV bands available for unlicensed broadband wireless devices [1], [2]. Spectrum utilization at any given location, frequency and time is highly inefficient and interest in allowing opportunistic users to access licensed spectrum has been tremendously growing among regulatory bodies (e.g. FCC in US and Ofcom in UK) and standardization groups such as IEEE802.16h, IEEE802.11af, P1900.4a etc.

Traditional orthogonal frequency division multiplexing (OFDM) system can be initially thought of as a natural choice for a multicarrier cognitive radio PHY. Although OFDM offers an efficient equalization and is very robust to frequency selective channels, it is not well suited for next generation cognitive radio waveform designs. In presence of carrier frequency offsets, the orthogonal subcarriers of OFDM are extremely vulnerable to interference and makes the entire OFDM system very sensitive. OFDM with rectangular pulse shaping has quite high out of band leakage which makes it unsuitable for cognitive radio applications in fragmented white space applications and the requirement for a cyclic prefix limits its spectral efficiency [3]. New multicarrier schemes are being investigated and among them filter bank multicarrier (FBMC) [4], [5] is extremely popular. FBMC with its prototype filter can satisfy regulatory requirements in its out of band leakage performance, but with no cyclic prefix, suffers from severe limitations in the synchronization front [6].

The multiband Generalized Frequency Division Multiplexing is a relatively new idea for designing a multicarrier PHY [7], [8]. GFDM is block based multicarrier transmission scheme derived from filter bank approach where the transmit data of each block is distributed in time and frequency and each subcarrier is pulse shaped with an adjustable pulse shaping filter. GFDM is well suited for cognitive radio, as the choice of pulse shaping filters makes the out-of-band leakage extremely small. Compared to OFDM, which has rectangular pulse shaping, GFDM with a choice of transmit pulse shaping, causes lesser interference to the adjacent incumbent frequency bands. This pulse shaping technique improves the adjacent channel leakage ratio (ACLR) of the GFDM system by around 20 dB. Another feature of GFDM is a tail biting cyclic prefix (CP). This feature is useful in cyclostationary detection [9], [10]. Compared to FBMC, which has no cyclic prefix, GFDM with its shortened CP, addresses the synchronization issues that was problematic in FBMC.

In an extremely fragmented white space cognitive radio scenario, a high ACLR is a necessity as it lowers the interference to incumbent signals in the adjacent bands. In OFDM the out of band leakage is around −13 dB; with raised cosine or root raised cosine pulses, the out of band leakage of a GFDM system is around −35 dB. To improve the adjacent channel leakage ratio even further, a technique of inserting cancellation carriers [11], [12] is implemented. Cancellation carriers are inserted at the vicinity of interference avoidance notch and are designed such that these cancellation carriers mitigate the interference from other subcarriers to the adjacent band. With this technique the out of band leakage is lowered to around −65 dB. This stunning improvement in the ACLR properties of GFDM satisfies stringent FCC requirements for cognitive radio transmissions in white space scenarios.

The rest of the paper is structured as follows: Section II describes the GFDM transmission model, followed by Section III where active interference cancellation is described. Next, performance comparison between different methods are
written in Section IV followed by conclusion in Section V.

II. GFDM TRANSMISSION MODEL

GFDM is a multi-carrier modulation scheme with flexible pulse shaping. Initially the binary data is modulated and divided into sequences of $K \times M$ complex valued data symbols. Each such sequence $d[\ell]$, $\ell = 0, 1, \ldots, KM - 1$, is spread across $K$ subcarriers and $M$ time slots for transmission. The data is represented by means of a block structure defined in equation (1) as

$$
D = \begin{bmatrix}
    d_0[0] & \ldots & d_0[M - 1] \\
    \vdots & \ddots & \vdots \\
    d_{K-1}[0] & \ldots & d_{K-1}[M - 1]
\end{bmatrix},
$$  

(1)

where $d_k[m] \in \mathbb{C}$ is the data symbol transmitted on the $k$th subcarrier and in the $m$th time slot. OFDM is a special case of GFDM, where $M = 1$, i.e. the data is pread only across frequencies and not in time.

The GFDM transmitter structure is shown in Fig. 1. In the $k$th branch of the transmitter, the complex data symbols $d_k[m]$, $m = 0, \ldots, M - 1$ are upsampled by factor $N$, resulting in

$$
d_k^N[n] = \sum_{m=0}^{M-1} d_k[m] \delta[n - mN], \quad n = 0, \ldots, NM - 1,
$$  

(2)

where $\delta[\cdot]$ is the Dirac delta function. Consequently, $d_k^N[mN] = d_k[m]$ and $d_k^N[n] = 0$ for $n \neq mN$.

The pulse shaping filter $g[n]$ is applied to the sequence $d_k^N[n]$, followed by digital subcarrier upconversion. The resulting subcarrier transmit signal $x_k[n]$ can be mathematically expressed as

$$
x_k[n] = (d_k^N \circledast g)[n] \cdot w^{kn}
$$  

(3)

where $\circledast$ denotes circular convolution and $w^{kn} = e^{j \frac{2\pi}{NM} n}$. Similar to equation (1), the transmit signals can be expressed in a block structure

$$
X = [x_0[0] \quad \ldots \quad x_0[MN - 1]]
= \begin{bmatrix}
    x_0[0] & \ldots & x_0[MN - 1] \\
    \vdots & \ddots & \vdots \\
    x_{K-1}[0] & \ldots & x_{K-1}[MN - 1]
\end{bmatrix}
$$  

(4)

The transmit signal for a data block $D$ is then obtained by summing up all subcarrier signals according to

$$
x[n] = \sum_{k=0}^{K-1} x_k[n].
$$  

(5)

As shown in [13], when all transmit samples are collected in a vector $x = [x_0, \ldots, x_{K-1}]^T$, the GFDM transmitter can be formulated as [8]

$$
x = Ad.
$$  

(6)

Herein, $A$ is a $NM \times KM$ complex valued modulation matrix with elements based on the parameters $M, K, N$ and $g[n]$.

The matrix $A$ contains all transmit signal processing operations and is given by

$$
x = W_N^H \sum_{k=0}^{K-1} P^{(k)}r^{(L)}_X R^{(L)}_M d_k,
$$  

(7)

where the data symbols $d_k$ on the $k$th sub-carrier are first transformed to frequency domain by multiplication with an $M \times M$ discrete Fourier transform (DFT) matrix $W_M = \{w_{i,j}\}_{M \times M}$, where $w_{i,j} = e^{-j2\pi i j}$. Then, tightly related to the localized non-zero coefficients of the frequency response of the pulse, the resulting frequency samples are duplicated $L$-fold by multiplication with a repetition matrix $R^{(L)} = (I_M \quad I_M \quad \ldots \quad I_M)$, which is a concatenation of $L$ identity matrices $I_M$ of size $M \times M$. This operation corresponds to an $L$ times upsampling in time domain.

Subsequently, each sub-carrier is filtered with $r^{(L)}_X$, a matrix which contains $R^{(L)}_M$ on its diagonal and zeros otherwise. Note that, while $g$ contains $NM$ filter coefficients, $g^{(L)}$ can be downsamples by $N/L$ and thus reduced to only $LM$ samples.
if it contains negligible filter coefficients that are zero and near-zero in frequency domain.

Finally, the \( k \)-th sub-carrier is up-converted to its respective sub-carrier frequency with the permutation matrix \( P^{(k)} \), which can be constructed according to

\[
P^{(0)} = \begin{pmatrix}
I_{LM/2} & 0_{LM/2} & \cdots & 0_{LM/2} & 0_{LM/2}
\end{pmatrix},
\]

\[
P^{(1)} = \begin{pmatrix}
0_{LM/2} & I_{LM/2} & \cdots & 0_{LM/2} & 0_{LM/2}
\end{pmatrix},
\]

etc. with \( 0_{LM/2} \) being an \( \frac{LM}{2} \times \frac{LM}{2} \) matrix containing zero elements. This matrix shifts and exchanges the upper and lower part of the base band spectrum of the sub-carrier onto its band pass representation.

After that, the signals of all \( K \) sub-carriers are super-positioned and the result is transformed back to the time domain with \( W^{NM} \).

### III. ACTIVE INTERFERENCE CANCELLATION

In this section, we explain the implementation of cancellation carrier insertion [11] [12] [14] in GFDM for out of band leakage reduction. Fig. 3 shows the subcarrier allocation of cancellation carriers inserted in GFDM system. CCs are inserted close to the interference avoidance notch and are calculated to suppress the side-lobes of the data subcarriers on the null band.

Instead of turning off a large number of subcarriers, we define two special subcarriers at the edge of the interference band as shown in Fig. 3. How to create the notch using the minimum number of subcarriers and how to compute the CCs are discussed below. It is shown in [11] that CCs play the dominant role in cancelling out the interference to the interference avoidance band, and the 'in-band' subcarriers can be simply turned off.

Let us consider the vector \( v_1 \) given by

\[
v_1 = Au,
\]

where \( A \) is the kernel defined by (6) and \( u \) is the vector of the data subcarriers with CC and the inband subcarriers turned off. To cancel the interference within the notch, the negative of the interference signal needs to be generated as shown in [11]

\[
A_1 h = -v_1,
\]

where \( h \) is the column vector of the CCs along with the inband zeroed-off carriers. \( A_1 \) is the small kernel derived from \( A \) considering the length of \( h \). With the help of Moore-Penrose generalized inverse, the minimum mean-squared solution of

\[
e^2 = \|A_1 h + v_1\|^2,
\]

is given as

\[
h = - (A_1^T A_1)^{-1} A_1^T v_1 = - Q_1 v_1
\]

\[
h = -Q_1 Au = -Q_2 u
\]

where \( Q_1 \) and \( Q_2 \) are pre-computable as the interference avoidance band is predefined.

The cancellation carrier vector \( h \) is then appended to the transmit data vector. For the ease of equalization at the receiver, a cyclic extension is appended to \( x[n] \) to obtain \( \tilde{x}[n] \). Tail biting [15], has been applied to GFDM and this has been used to reduce the length of the CP. It is used to maintain the

![Fig. 2. GFDM receiver system model with interference cancellation block](image-url)

![Fig. 3. Sub-carrier allocation for cancellation carrier insertion](image-url)
circular structure within each block. This tail biting concept exploits the digital implementation of the filters to perform circular convolution. $\tilde{x}[n]$ is then passed to the digital-to-analog converter and sent over the channel.

The work in this paper is on improving ACLR characteristics by modifying the transmitter. The fundamental concept of the GFDM receiver, shown in Fig. 2, remains unchanged. The GFDM receiver model with self interference cancellation block is described in details in [8].

IV. ACLR PERFORMANCE

Adjacent channel leakage ratio is an important metric in choosing the next generation physical layer modulation scheme for cognitive radio networks. In OFDM, because of rectangular pulse shaping, the out of band leakage is quite high at $-13$ dB. Insertion of cancellation carriers in conjunction with time domain windowing has been implemented for OFDM [12]. For this interference avoidance partition frequency technique (IAPFT) method, the ACLR is about $-40$ dB. The comparison between the two is shown in Fig. 4 taken from [16], where a comparative analysis of OFDM, nominal GFDM with RRC pulse and IAPFT transmission schemes has been studied by the authors. It has been shown that cancellation carrier insertion lowers the out of band leakage of an OFDM system by around $30$ dB. This motivates the study and implementation of CC insertion in a GFDM system.

GFDM with the flexibility of pulse shaping can achieve lower out of band leakage of around $-35$ dB. In this work, raised cosine and root raised cosine pulses were used to shape the GFDM symbols per subcarrier-wise. Active interference cancellation is implemented by inserting 2 cancellation carriers on either side of the interference avoidance notch. As shown in Fig. 5, with insertion of cancellation carriers, the out of band leakage of the GFDM system is now around $-65$ dB. This improvement comes at a cost of $2$ subcarriers (CCs) on either side of the interference avoidance notch. This decreases the spectral efficiency in terms of subcarriers used to carry useful data, but improves the ACLR of the GFDM system significantly and satisfies the stringent regulatory ACLR requirement of $-65$ dB for opportunistic signals in TV white space. The simulation parameters are provided in Table 1.

V. CONCLUSIONS

GFDM is an extremely attractive multicarrier modulation scheme suitable for cognitive radio PHY as it has a low out-of-band radiation into the adjacent frequency bands. Compared to OFDM, nominal GFDM has higher ACLR figures. But $-35$ dB out of band leakage does not make GFDM suitable to be used as an opportunistic waveform in TV white spaces. With insertion of cancellation carriers, GFDM out of band leakage is now $-65$ dB. It now can satisfy strict FCC regulatory requirement of out of band leakage for opportunistic dynamic access of TV whitespace. The work presented here compares other similar methods implemented for OFDM and shows that GFDM out performs all and matches ACLR performance of FBMC. This paper presents significant achievement in improving the ACLR properties of GFDM and hence highlights it as a suitable candidate for a 5G flexible PHY in fragmented TV white space cognitive radio transmissions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
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<tr>
<td>Modulation scheme</td>
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<td>Samples per symbol</td>
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<tr>
<td>Subcarriers</td>
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<td>Cancellation Carriers</td>
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<tr>
<td>Roll-off factor</td>
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TABLE I
GFDM SIMULATION PARAMETERS.
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