Abstract—Existing non data-aided synchronization schemes for OFDM in particular exploit the redundancy implemented by the CP to perform a time and frequency offset estimation. In general this is more challenging to exploit in non-orthogonal multicarrier systems. This paper gives a short insight of the impact of frequency offset of the performance of the non-orthogonal scheme GFDM and proposes a new estimator for it, which makes use of the data repetition implemented by the sidelobes of the prototype filter and is not feasible in OFDM. It allows a coarse CFO estimation in a wide range, even in a tough multipath environment, which can be improved by using a larger number of data.

I. INTRODUCTION

The throughput of cellular systems increased from a few kB/s in the 2nd generation to a hundred Mb/s promised in the 4th generation. The demand is still increasing as new applications emerge and consequently further growth of data traffic can be projected for the future. As an example, the current trends indicate that media content offered by radio and TV will be delivered through Internet protocol networks even more in the future, as people want to enjoy their favorite songs and movies individually anywhere and at any time. Orthogonal Frequency Division Multiplexing (OFDM) is a today’s state of the art multicarrier transmission technique and widely used in modern communication systems. OFDM benefits from the advantage of multicarrier systems in frequency selective channels, because the data is transmitted over a set of subcarriers, whose bandwidth is small enough to be considered frequency flat in contrast to the overall system bandwidth. Further, in OFDM the subcarriers are orthogonal, eliminating inter carrier interference (ICI). Nevertheless the implicit filtering of the subcarriers with a rectangular function in time domain is known to create strong out-of-band (OOB) radiation, which makes it unfavorable when considering fragmented spectrum. An additional drawback is high peak-to-average power ratio, which has implications for the quality of amplifier components. Lastly, the parametrization of the scheme exhibits limited flexibility of parameters making it less useful for future cellular scenarios. Alternative waveforms, promising to better fulfill the requirements of future mobile communication systems, are being investigated in [1], [2].

Among them, Generalized Frequency Division Multiplexing (GFDM) [2], [3] is a newly researched scheme, which is considered as a candidate for 5G mobile communication. It addresses the requirements mentioned above with additional degrees of freedom regarding the number of sub-symbols per subcarrier and the subcarrier pulse shape compared to OFDM. That way OOB radiation and peak-to-average ratio (PAPR) can be controlled and depending on the use case, time and frequency resources can be partitioned in a flexible way. The cost for this are inter carrier interference (ICI) and inter symbol interference (ISI) which results from a possibly non-orthogonal subcarrier filter. Nevertheless, certain iterative and non-iterative receiver structures can mitigate the impact of interference on the GFDM performance [4]. Also, implementation is possible using the fast Fourier transform (FFT) algorithm, which shows that it can compete with established solutions in terms of complexity [5].

A challenge in multicarrier schemes is frequency synchronization. Due to the enormous popularity of OFDM, a vast number of approaches are available for this scheme. Most of them are based on data-aided algorithms to provide a good estimation as fast as possible [6]–[8]. Also non data aided solutions, exploiting the redundancy introduced by the cyclic prefix (CP) are existent [9], [10]. The algorithms of OFDM can not be directly implemented in GFDM, because of the self created interference. Thus, for GFDM only a few methods have been proposed, so far exclusively based on isolated training sequences [11].

This paper presents a non data-aided scheme that can exploit the self generated ICI for synchronization and thus benefits from this property, which is actually undesired. This approach cannot be implemented in an orthogonal scheme such as OFDM. The proposed scheme does not use any training sequence and thus does not affect the out-of-band properties. An initial synchronization can be obtained via a faster data-aided scheme. The rest of this paper is organized as follows: Section 2 describes the GFDM system in general. Section 3 gives a short insight of the impact of frequency offset on the system. Section 4 shortly draws an existing synchronization scheme for OFDM and describes the proposed method detailed in contrast, while Section 5 presents simulative results to show the performance of the approach. Finally, Section 6 concludes this paper by providing a summary.

II. GFDM SYSTEM DESCRIPTION

The time and frequency signal properties of the waveform in GFDM can be engineered according to specific application requirements. While both schemes follow a block based approach, in OFDM each multi carrier symbol attaches one cyclic prefix (CP), while in GFDM a group of M multi carrier symbols is jointly prefixed. Hereby a multi carrier symbol consists of K subcarriers with N = KM samples. Out of this, K_{in} subcarriers will be used for data transmission, while the remaining subcarrier are nulled. Additionally, GFDM allows to choose a pulse shaping filter for each subcarrier. The differences between the block definition of OFDM and
GFDM is pictured in Fig. 2.

The block diagram of the GFDM transmitter and receiver is depicted in Fig. 1. In order to transmit and receive one block of data, first a data source produces a sequence of bits $b[\ell]$, $\ell = 0, \ldots, \mu(K_{on}M - 1)$. These are then mapped to complex data symbols $\tilde{d}[\ell]$, $n = 0, \ldots, N - 1$, which are drawn from a $\mu$-ary constellation. Thereby the last $(K - K_{on})M$ symbols are put to zero. This results in an $K/K_{on}$ oversampling in time domain at the output at the transmitter. In the GFDM modulator block, a sequence of $N$ data symbols is split into $K$ groups of $M$ symbols, yielding $d_k[m]$ with $k = 0, \ldots, K - 1$ denoting the subcarrier index and $m = 0, \ldots, M - 1$ representing a lower rate sub-symbol index. Each $d_k[m]$ is transmitted with a corresponding pulse shape

$$g_k,m[n] = g[(n - mK) \mod N] \cdot e^{j2\pi \frac{k\ell}{N}}, \quad (1)$$

which is based on a time and frequency domain circular shift of a prototype filter $g[n]$. Using this, the transmit samples $x[n]$ are then obtained according to

$$x[n] = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} d_k[m]g_k,m[n]. \quad (2)$$

In the last processing step on the transmitter side, a CP of $N_{CP}$ samples is added to the block, yielding

$$\tilde{x}[n] = \{x[N - N_{CP}], \ldots, x[N - 1], x[0], \ldots, x[N - 1]\}. \quad (3)$$

Note that the range of the sample index of the prefixed signal is extended to $n = -N_{CP}, \ldots, N - 1$.

Transmission over a wireless channel can be modeled as the convolution

$$\tilde{y}[n] = (h[n] * \tilde{x}[n - \theta]) e^{j2\pi \epsilon n/N} + \tilde{w}[n], \quad (4)$$

with $\tilde{y}[n]$ denoting the samples of the received signal, $h[n]$ depicting a channel impulse response, $\theta$ denotes timing mismatch between transmitter and receiver, $\epsilon = \Delta f / f_{\infty}$ is the carrier frequency offset (CFO), normalized to the subcarrier frequency span and $\tilde{w}[n]$ denotes the additive white Gaussian noise (AWGN).

After receiving the signal samples $\tilde{y}[n]$, time synchronization is applied first, in order to obtain $\hat{y}[n]$. Then the CP is removed, which yields $y[n]$. Subsequently, CFO is compensated and the synchronized signal $y[n]$ is obtained. In the next step, the effect of the channel impulse response is reverted with frequency domain equalization (FDE), which yields the equalized signal $\hat{y}[n]$.

In order to demodulate the GFDM signal, several receiver structures can be utilized. The matched filter receiver (MF) can be used to maximize the signal-to-noise ratio per subcarrier. It is given as

$$\hat{d}_k[m] = (y[n] \otimes g^*_{k,m}[-n])|_{n=0}. \quad (5)$$

This approach however suffers in error rate performance, when the prototype pulse $g[n]$ is chosen such, that orthogonality between subcarriers and sub-symbols cannot be maintained and self-interference arises [12]. The impact of self-interference can be prevented with the zero forcing (ZF) receiver [4]. It inverts the effect of the modulation, however is known to suffer from noise enhancement.

Lastly, using a linear minimum mean square error (MMSE) receiver [4], the variance of the AWGN can be taken into account in order to improve robustness.

In the last step of the receiver chain, the transmitted bits $\hat{b}[\ell]$ are recovered from the received data symbols $\hat{d}_k[m]$ with a demapper. The parameters, which are used for the rest of the paper are specified in Tab. I. As prototype filter a raised cosine with parameter $\alpha$ is used, with $\alpha$ defining the steepness.

<table>
<thead>
<tr>
<th>setup</th>
<th>$K$</th>
<th>$K_{on}$</th>
<th>$M$</th>
<th>$\alpha$</th>
</tr>
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<tr>
<td>channel</td>
<td>1st tap</td>
<td>2nd tap</td>
<td>3rd tap</td>
<td>$\tau_{rms}$</td>
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<td>power delay profile</td>
<td>0 dB</td>
<td>-2.5 dB</td>
<td>-5 dB</td>
<td>5.1ms</td>
</tr>
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</table>

TABLE I: Used parameters set. $K$ denotes the number of subcarriers, $K_{on}$ the number of used subcarriers, $M$ represents the number of multicarrier symbols per GFDM block. A Rayleigh fading channel is used to simulate the influence of multi path transmission.
III. SYNCHRONIZATION

In a wireless communication system, a frequency offset between transmitter and receiver can occur in consequence of mismatch or drift between oscillators. This is even more relevant, when low budgeted components are used. The goal of this section is to investigate the impact of the CFO to the performance of the system and to give an overview over an exiting synchronization algorithm for OFDM and finally propose a scheme to achieve synchronization in GFDM.

A. Impact of Carrier Frequency Offset

Frequency offset introduces additional interference to the signal. Also the mean of the probability density function (pdf) of the received data symbols is shifted which has further negative influence on the detection quality. Note that for the moment, the wireless channel is neglected, and a phase synchronization at the beginning of every block is applied in order to isolate the performance degradation due to the mismatch. The influence of the CFO to the pdf of a BPSK constellation is illustrated for an exemplary configuration in Fig. 3. Compared to OFDM the self interference in GFDM causes a larger spread of the pdf. This is caused by the fact, that the OFDM block is about factor M shorter than a GFDM block, which results in a much higher phase offset at the end of the GFDM block than in an OFDM block, giving a larger shift of the signal mean and higher interference spread.

After showing the isolated impact of the offset in the pdf, we now consider a more realistic scenario using a multipath channel. Assuming that a BER of $10^{-2}$ is a reasonable number for transmission without error correction coding, the signal-to-noise ratio (SNR), which is necessary to achieve this error rate with a given frequency offset can be used as an indicator for performance. In Fig. 4, simulations show that the maximum frequency offset that can be tolerated for GFDM is $\epsilon = 0.03$ for QPSK and $\epsilon = 0.01$ for 16 QAM. OFDM has a different performance, mainly because of the smaller block length. This penalty of GFDM compared to OFDM emphasizes the need of a good synchronization mechanism in GFDM.

B. Existing Synchronization Algorithm for OFDM

Among the different approaches for synchronizing OFDM, several methods are existing, which exploit data redundancy implemented by the CP to get statistical information about time and frequency shift. One of them will be presented here.

In [10], Beek et al. proposed a joint maximum likelihood estimator (MLE) for time and frequency estimation. The MLE is asymptotically the minimum variance unbiased estimator (MVUE) with growing iterations for an AWGN channel. Based on the assumption that the data values $d_k[m]$ are independent, he states that $x[n]$ after the IDFT is approximately complex Gaussian distributed, with real and imaginary values being independent. This process is not white due to the existence of the CP, yielding a correlation between some pairs of samples, which are located $K$ samples apart. He derived the estimator to be

$$\hat{\theta} = \arg\max_{\theta} \left\{ |\gamma| - \rho \Phi[\theta] \right\}$$  \hspace{0.5cm} (6)

$$\hat{\epsilon} = \frac{1}{2\pi} \angle \left\{ \hat{\theta} \right\}$$  \hspace{0.5cm} (7)

with

$$\gamma[m] = \sum_{k=m}^{m+L_{cp}-1} y[n] y^*[n+K]$$  \hspace{0.5cm} (8)

$$\Phi[m] = \frac{1}{2} \sum_{k=m}^{m+L_{cp}-1} |y[n]|^2 + |y[n+K]|^2$$  \hspace{0.5cm} (9)

$$\rho = \frac{\sigma_z^2}{\sigma_s^2 + \sigma_n^2}.$$  \hspace{0.5cm} (10)

In this approach, the tolerated offset is limited to being residual.

In non-orthogonal schemes, the exploitation of this redundancy introduced by the CP in time domain is more challenging, because the filter distorts the signal and creates dependency between the multicarrier symbols within the block. In addition, only one CP per $M$ multicarrier symbols is distributed, while in OFDM $M=1$.

Besides the CP, GFDM has another source of redundancy, which can be used for synchronization. If the bandwidth of the used filter is greater than the subcarrier bandwidth, some parts of the spectrum are repeated. This redundancy in frequency domain can be used for synchronization.
redundancy is in frequency domain, the DFT of the signal has to be calculated in order to evaluate this information. This can only be done correctly, if a time synchronization is applied before. The $MK$ point DFT of the received samples with known time offset $\theta$ can be expressed as

$$Y[l] = \text{DFT} \{y[\theta + n]\}$$  \hspace{1cm} (14)$$

As it is known that the redundant parts in frequency domain have a distance of $M$ samples, the correlation of the two redundancy parts of the signal can be calculated in analogy to (8) as

$$\gamma_i[l] = \sum_{k=1}^{V} Y[l + k] \cdot Y^*[l + k + M]$$  \hspace{1cm} (15)$$

with

$$l = 0, \ldots, MK - 1 \quad V = \lceil M/2 \rceil.$$  

Here, $V$ is the number of samples in frequency domain used for correlation and $i = 1, \ldots, P$ equals the number of the GFDM block. From this follows, that the resolution of the possible estimated value is limited by the $KM$ data points in frequency domain. Averaging $\gamma_i[l]$ over all $P$ blocks gives

$$\bar{\gamma}[l] = \frac{\sum_{i=1}^{P} \gamma_i[l]}{P}.$$  \hspace{1cm} (16)$$

The averaged correlation produces $K_{\text{on}}$ peaks in the metric $\bar{\gamma}[l]$, one for every active subcarrier.

To use the statistical information of all peaks, a new metric is proposed. It uses the fact, that the distance between the peaks equals $M$ samples, which means that every subcarrier produces a peak in frequency domain. The proposed metric is given with

$$\Psi[d] = \sum_{k=0}^{K_{\text{on}}-1} |\gamma'[d + kM]|$$  \hspace{1cm} (17)$$

with $d = 0, \ldots, MK$ and $\gamma'[l]$ is assumed to have a repeated sequence concatenated, so that $\gamma'[l'] = \gamma'[l'] \mod MK - 1$ with $l' = 0, \ldots, 2 \cdot (MK - 1)$. The metric $\Psi[d]$ adds up all samples in distance $M$ of the metric $\gamma'[l']$ for one value of $d$. Consequently, the frequency shift can be derived as

$$\hat{\epsilon} = \frac{\arg\max \{\Psi[d]\} - \lceil V/2 \rceil}{M}. \hspace{1cm} (18)$$

The range of the estimation is in general is $-K/2, \ldots, \epsilon, \ldots, +K/2 - 1/M$. Because huge errors can occur at the border, the range is limited to $\Delta_\epsilon = -K/2 + 1, \ldots, \epsilon, \ldots, +K/2 - 1$. The accuracy is limited to be a multiple of $1/M$, caused by the fact, that there are $M$ sampling points per subcarrier in frequency domain. The metric can be enhanced, if $K_{\text{on}} < K$, i.e. an oversampling is applied.

### IV. Simulations

For simulation, a GFDM setup according to Tab. I was chosen with the motivation to have a minimal set of subcarriers. This setup is flexible and can use a small gap in the spectrum and thus is applicable for sensor utilization. The used raised
cosine filter with roll-off factor $\alpha = 1$ produces large side lobes in frequency and thus minimum redundancy of data. As performance criteria the mean square error (MSE) \cite[p.19]{13} has been chosen. It is equal to the variance in the case of an unbiased estimator with $b = 0$ and a common quality indicator $\Psi[d]$. The frequency offset $f_{sc}$ was be fixed to be multiples of $1/M$ to offer the possibility of perfect synchronization.

As can be seen in Fig. 6, the best performance can be achieved in an AWGN channel with an accuracy of almost $\text{MSE} = 10^{-3}$, if 20 GFDM blocks are evaluated. The simulations also show, that a coarse frequency estimation in the proposed tough multipath environment is possible, if a minimum of 20 blocks is evaluated. A $\text{MSE} = 6 \cdot 10^{-2}$ can be achieved in this case. By increasing the evaluation time to 200 blocks, also a $\text{MSE} = 4 \cdot 10^{-3}$ can be achieved. Although the performance is much worse than the performance of $\text{MSE} = 2 \cdot 10^{-5}$ of the data aided method in this SNR range in \cite{5}, it can be used as coarse CFO estimator for streaming applications. The relatively high MSE compared to this proposal on the one hand is caused by the limited resolution of $1/M$ for the detectable frequency offset, on the other hand this is caused by the wide range of $\Delta_f$. Also in this case, only the properties of the spectrum of the signal is used and a very tough multipath scenario was chosen to simulate requirements of a cellular system. A Rayleigh channel causes fading in frequency domain, hence the correlation of the data is affected, which results in a generally higher MSE compared to the AWGN channel.

The performance of the estimator can be improved by raising the amount of blocks to be considered in the estimation. This gives a more reliable metric $\Psi[d]$ because more redundancy can be used, which consequently raises the achievable performance. One decade of more GFDM blocks results in a decade of lower MSE in using the proposed multipath channel. Consequently, this method is a possible low effort method for streaming applications, without effecting spectral efficiency. Also this synchronization information, achieved out of the spectrum of the signal could be combined with other synchronization methods, to enhance the MSE.

V. Conclusion

In this paper, a novel non-data aided method, for coarse frequency offset synchronization in GFDM has been proposed, without affecting the spectral efficiency. The redundant signal that leaks from the sidelobes of the prototype filter into adjacent subcarriers of the system is exploited. The estimation can be done while transmitting payload data. The spectral efficiency is not affected. The proposed method can be used in tracking CFO changes after an initial preamble based estimation. Simulations show, that an acquisition of 20 symbols is sufficient to get useful results in a very tough multipath scenario. Also the wide estimation range of a CFO of $-K/2 + 1$ to $+K/2 - 1$ is important aspect in comparison to other approaches, which allows only fractional CFO estimation. The performance of the estimation can be improved by using larger number of GFDM blocks, e.g. in streaming applications.

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